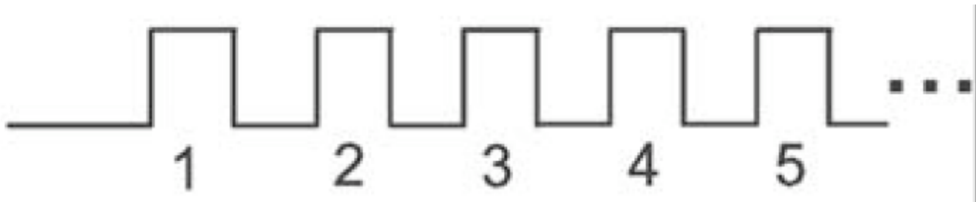


$$i\frac{dE_1}{dz} + \alpha E_1 + E_2 + \gamma |E_1|^2 E_1 = 0,$$

$$i\frac{dE_n}{dz} + \alpha E_n + (E_{n+1} + E_{n-1}) + \gamma |E_n|^2 E_n = 0$$

Stationary mode: $E_n(z) = \exp(i\beta z) E_n$

γ=0 : $E_n \sim \sin(kn)$ $\quad \beta = \alpha + 2\cos(k)$ $\quad k = \dfrac{m\pi}{N+1}$

NO SURFACE MODE

Surface Solitons

$$\gamma \neq 0$$

Use Newton-Raphson + judicious initial condition (antiadiabatic limit

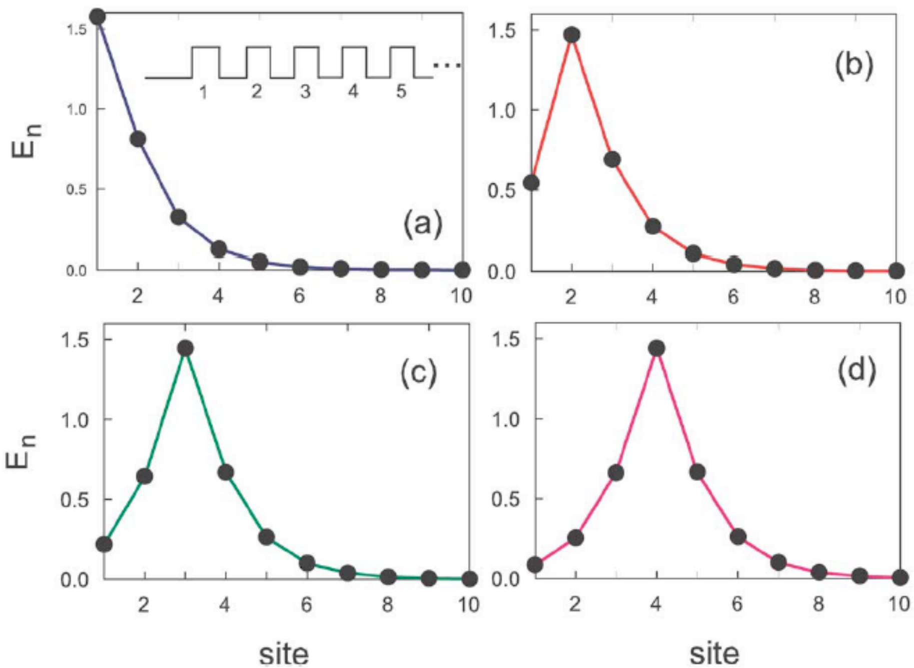


Fig. 1. (Color online) Examples of surface localized modes at $\beta = 3$ in an array of focusing waveguides ($\gamma = +1$) centered at distances d of (a) 0, (b) 1, (c) 2, (d) 3 from the array edge.

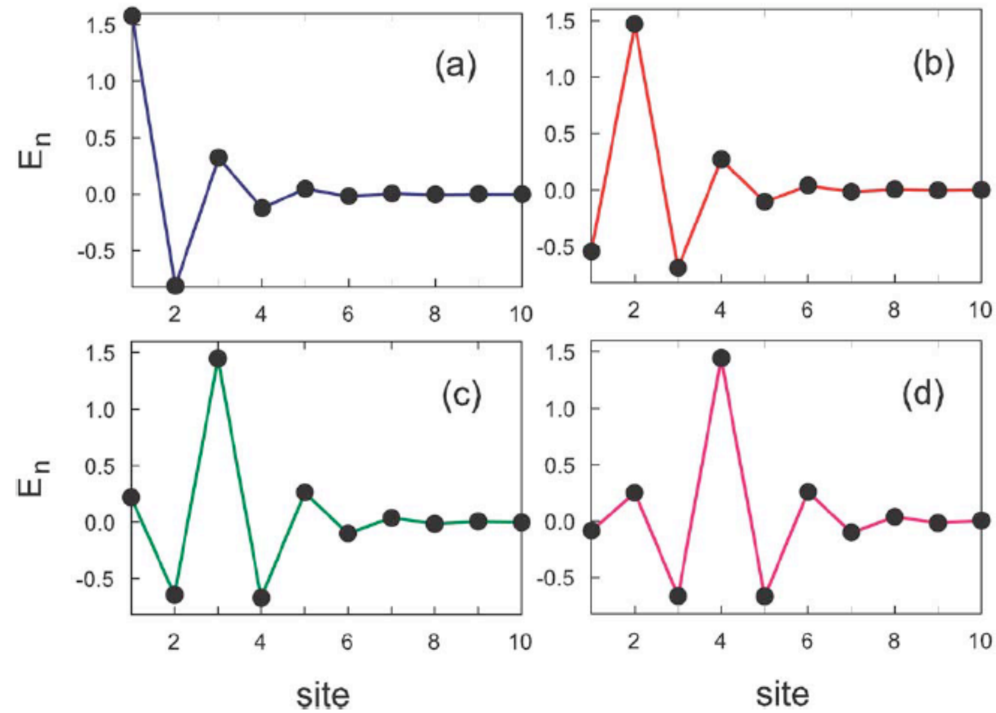


Fig. 2. (Color online) Examples of localized surface modes at $\beta = -3$ in an array of defocusing waveguides ($\gamma = -1$) located at distances d of (a) 0, (b) 1, (c) 2, (d) 3 from the array edge.

Surface Solitons

$$\text{Power} = \sum_n |E_n|^2$$

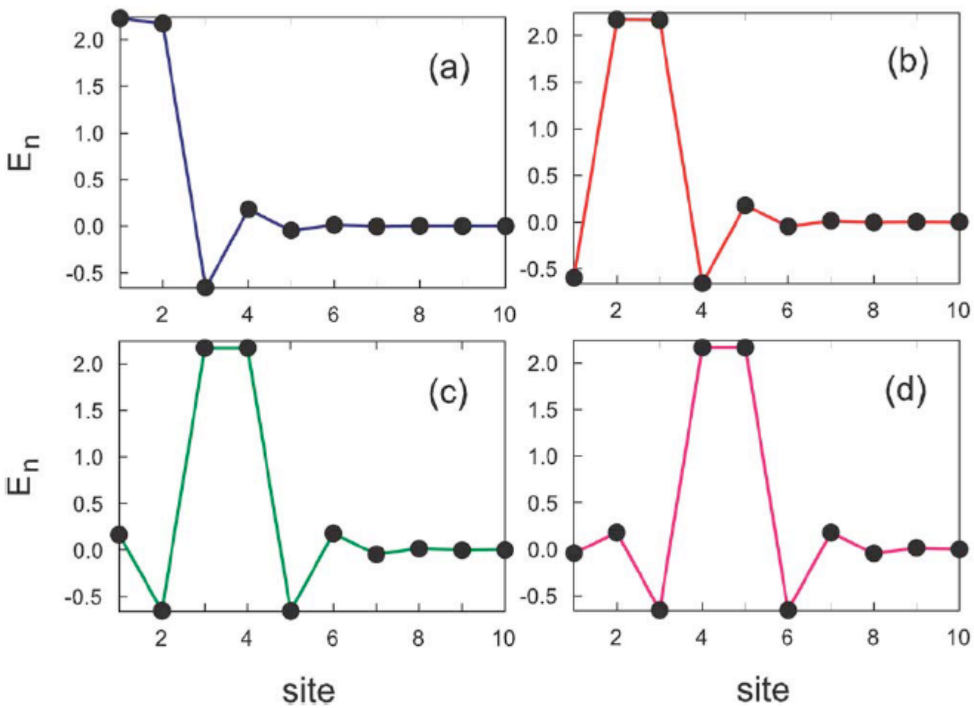


Fig. 5. (Color online) Examples of stable flat-topped localized surface modes at $\beta = -4$ in the array of defocusing waveguides ($\gamma = -1$) centered between various sites near the edge.

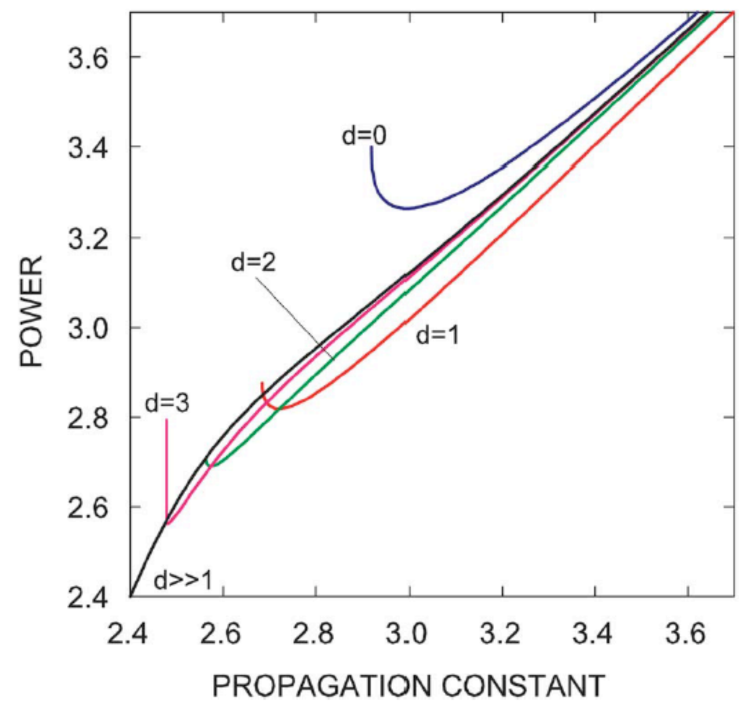


Fig. 3. (Color online) Normalized power versus propagation constant β for the surface modes shown in Fig. 1 located at distances $d = 0, 1, 2, 3$ from the surface. The darkest curve corresponds to the discrete soliton in an infinite array.

Surface Solitons

Existence and stability: The constraint method

$$H = -\sum_n (E_n E_{n+1}^* + E_n^* E_{n+1}) - (1/2) \sum_n |E_n|^4$$

$$X = \sum_n n|E_n|^2 / \sum_n |E_n|^2$$

$E_n \neq 2$

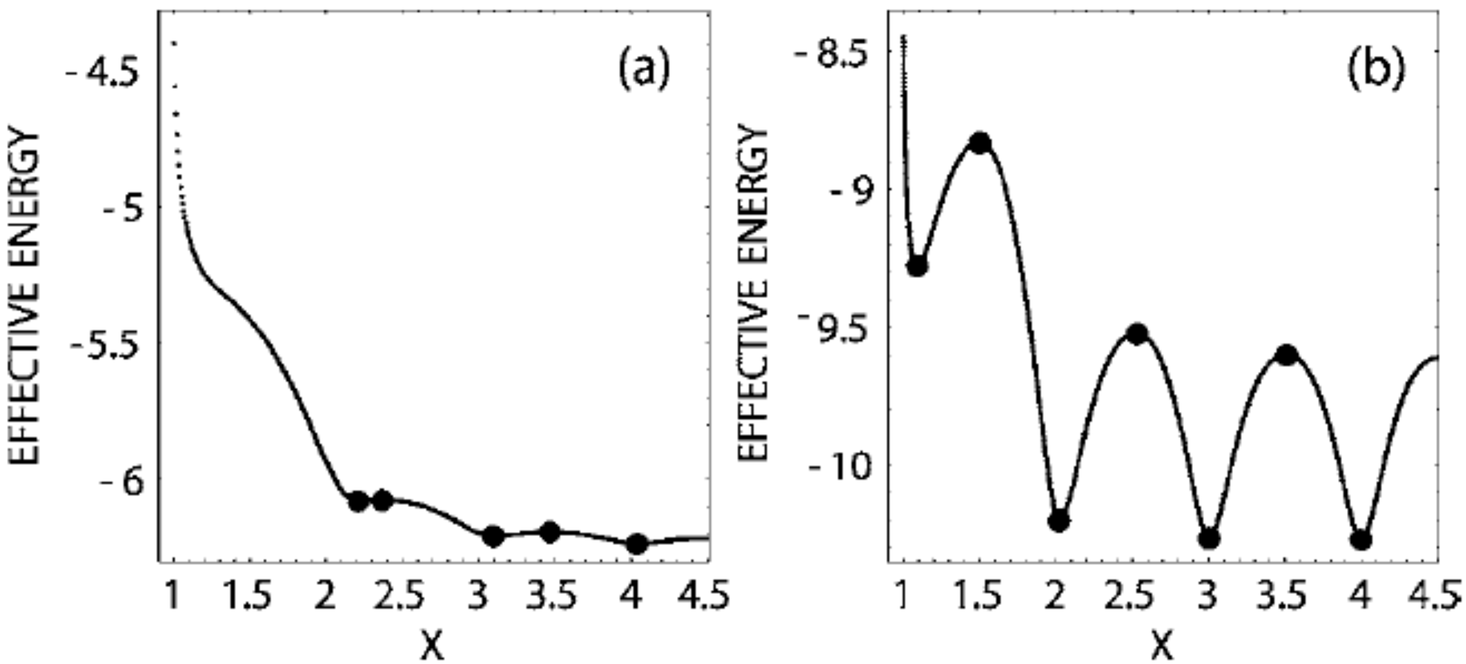
(1) Compute an odd mode centered at n. Obtain all $\{E_n\}$ and power P
(2) Fix amplitude at n+1 to be $E_{n+1} + \epsilon$
(3) Solve all NR equations for E_m $(m \neq n+1)$ keeping power fixed at P, arriving to intermediate state centered between n and $n+1$.
(4) Obtain X and H for intermediate state.
(5) increase ϵ and repeat procedure until amplitudes at sites n and $n+1$ coincide (even mode).

$$U_{\text{eff}} = H(X)$$
$$dH/dX = 0$$

Stationary solutions

Surface Solitons

$P < P_c$ $P > P_c$



$P < P_c$: surface is repulsive

even modes unstable

Surface solitons

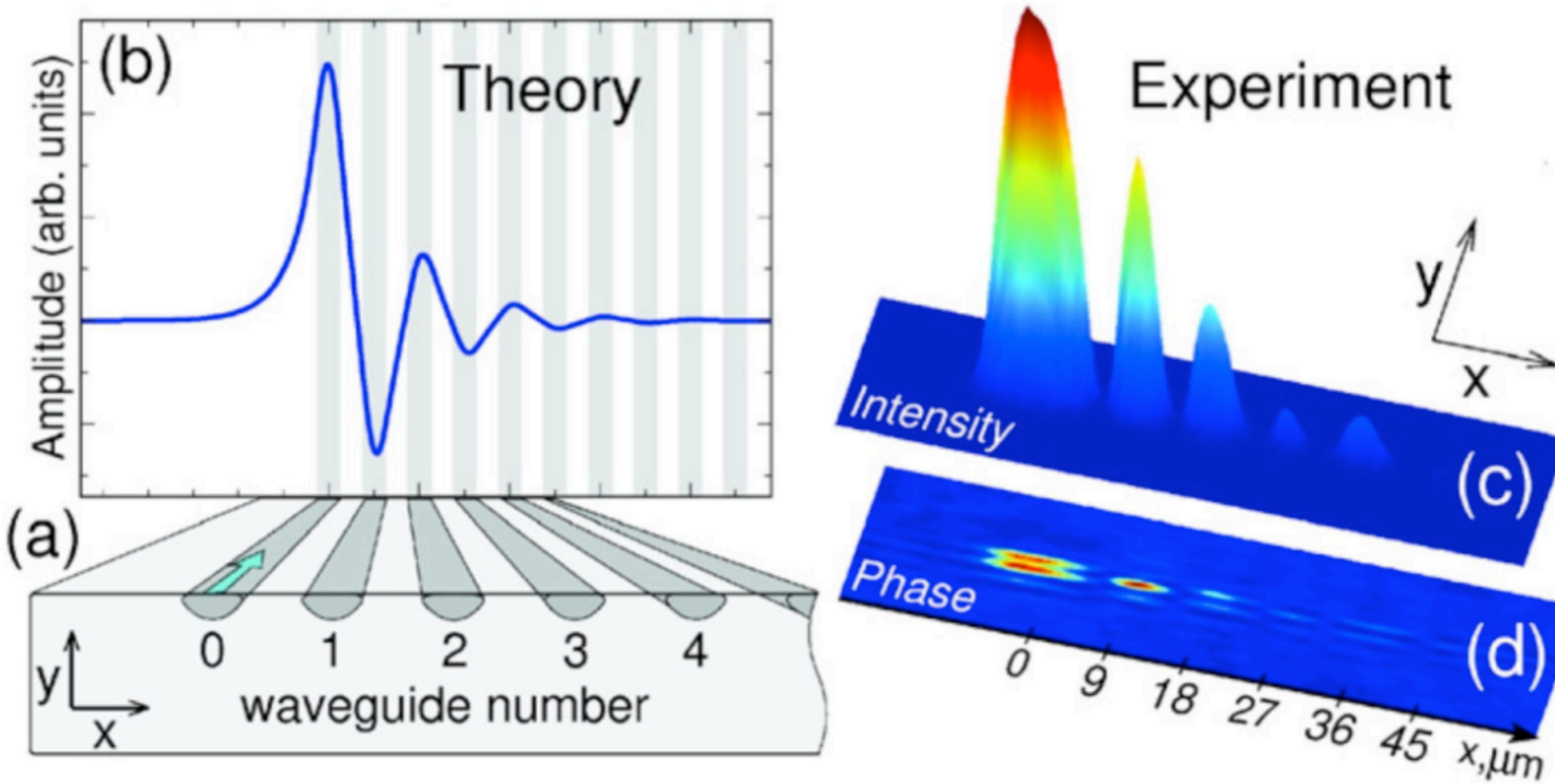


Fig.2: Theoretical prediction (a, b) and experimental observation (c, d) of nonlinear Tamm states in a truncated photonic lattice. (a) Schematic of the waveguide array geometry; (b) theoretical profile of a nonlinear Tamm statea surface gap soliton. (c) three-dimensional representation of the nonlinear surface state observed above the localization threshold. (x,y) are the horizontal and vertical sample coordinates, respectively. (d) Experimental plane–wave interferogram demonstrating the staggered phase structure of the nonlinear Tamm state (from M. I. Molina and Y. S. Kivshar, ref [14]). **PRL 97, 083901 (2006)**