Saturable impurity in an optical array: Green function approach

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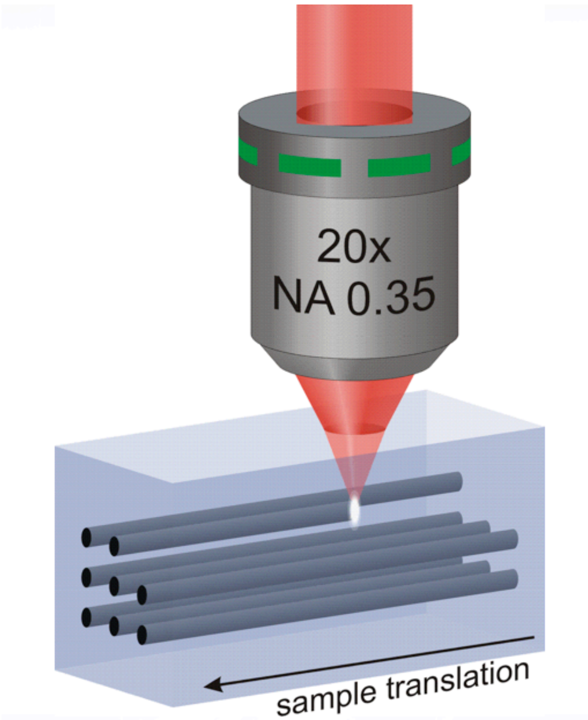
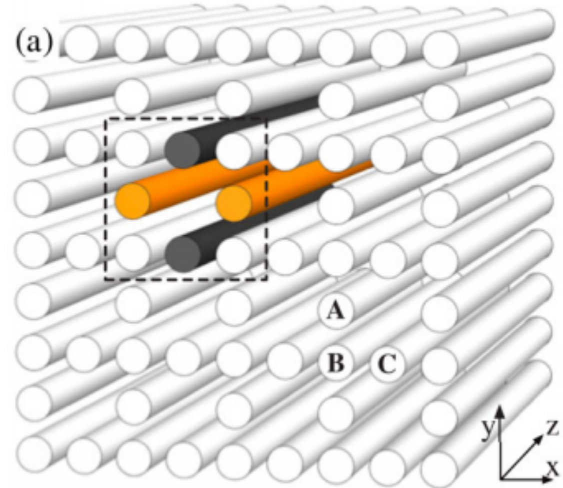
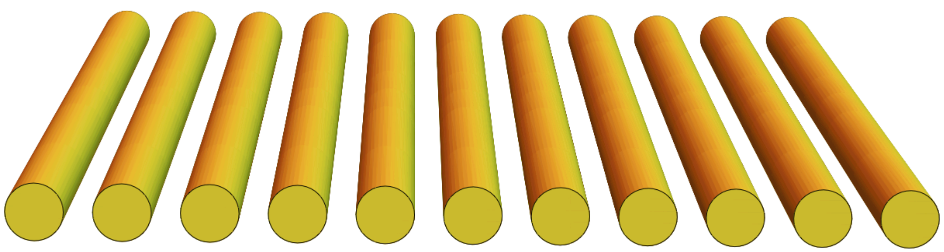
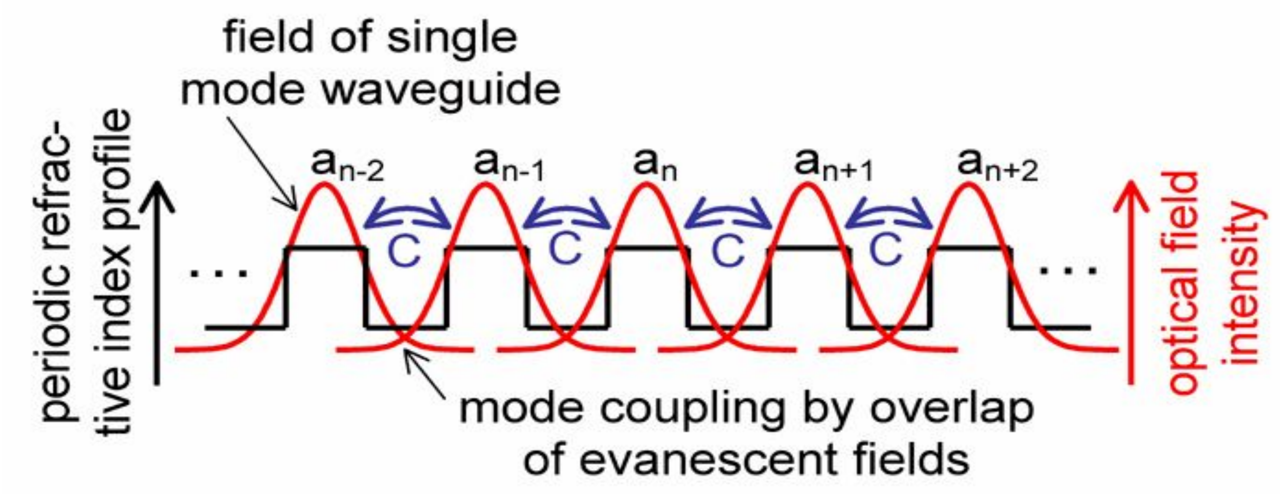
Optical waveguide arrays







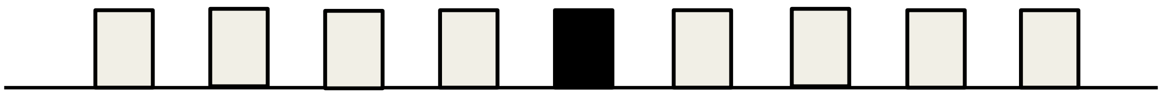


$$E(x,t) = \sum_n E_n(t)\ \phi(x-n)$$

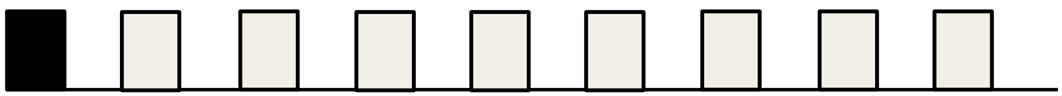
Coupled-modes

$$i\left(\frac{dE_n}{dz}\right) + V(E_{n+1} + E_{n-1}) + f(E_n)E_n = 0$$

Saturable impurity: $$f(E_n) = \chi\ \delta_{n,n_0}\ \left(\frac{1}{1+|E_n|^2}\right)$$



Bulk impurity



surface impurity

What do we want to compute?

Form of the localized mode at the impurity site and transmission of plane waves across the impurity in **closed form**

The Hamiltonian !!

$$\tilde{H} = \tilde{H}_0 + \tilde{H}_1$$

$$\tilde{H}_0 = V \sum_{nn} (|n\rangle\langle m| + h.c.)$$

$$\tilde{H}_1 = \frac{\chi}{1 + |E_d|^2} |d\rangle\langle d|$$

GREEN function $G(z) = \frac{1}{z - \tilde{H}}$

Poles of Green function → energies of bound states
Residues at poles → bound state amplitudes

Perturbative Expansion

$$G = G^{(0)} + G^{(0)} H_1 G^{(0)} + G^{(0)} H_1 G^{(0)} H_1 G^{(0)} + \cdots$$

$$G^{(0)} = 1/(z - H_0) \text{ and } H_1 = \gamma/(1 + |E_d|^2)$$

$$G_{mn} = G^{(0)}_{mn} + \frac{\varepsilon}{1 - \varepsilon\, G^{(0)}_{dd}}\, G^{(0)}_{md} G^{(0)}_{dn}$$

$$G_{mn} = \langle m|G|n\rangle \text{ and } \varepsilon = \gamma/(1 + |E_d|^2)$$

Bound state equation

$$1 = \varepsilon\, G^{(0)}_{dd}(z_b) = \gamma\, \frac{G^{(0)}_{dd}(z_b)}{1 + |E^{(b)}_d|^2}$$

$$G_{nd}^{(0)}(z) = \left(\frac{sgn(z)}{\sqrt{z^2-1}}\right)\left\{z - sgn(z)\sqrt{z^2-1}\right\}^{|n-d|}$$

$$|E_n^{(b)}|^2 = Res\{G_{nd}\}_{z=z_b} = -\frac{G_{nd}^{(0)2}(z_b)}{G'^{(0)}_{dd}(z_b)}$$

Bound state amplitudes

$$\frac{1}{\gamma} = \frac{z}{z^2 - 1 + |z|\sqrt{z^2-1}}$$

Energy equation

Analytic solution

$$z_b = -\left(\frac{1-\gamma^2}{6\gamma}\right) + \frac{1+10\gamma^2+\gamma^4}{6\gamma D(\gamma)} + \frac{D(\gamma)}{6\gamma}$$

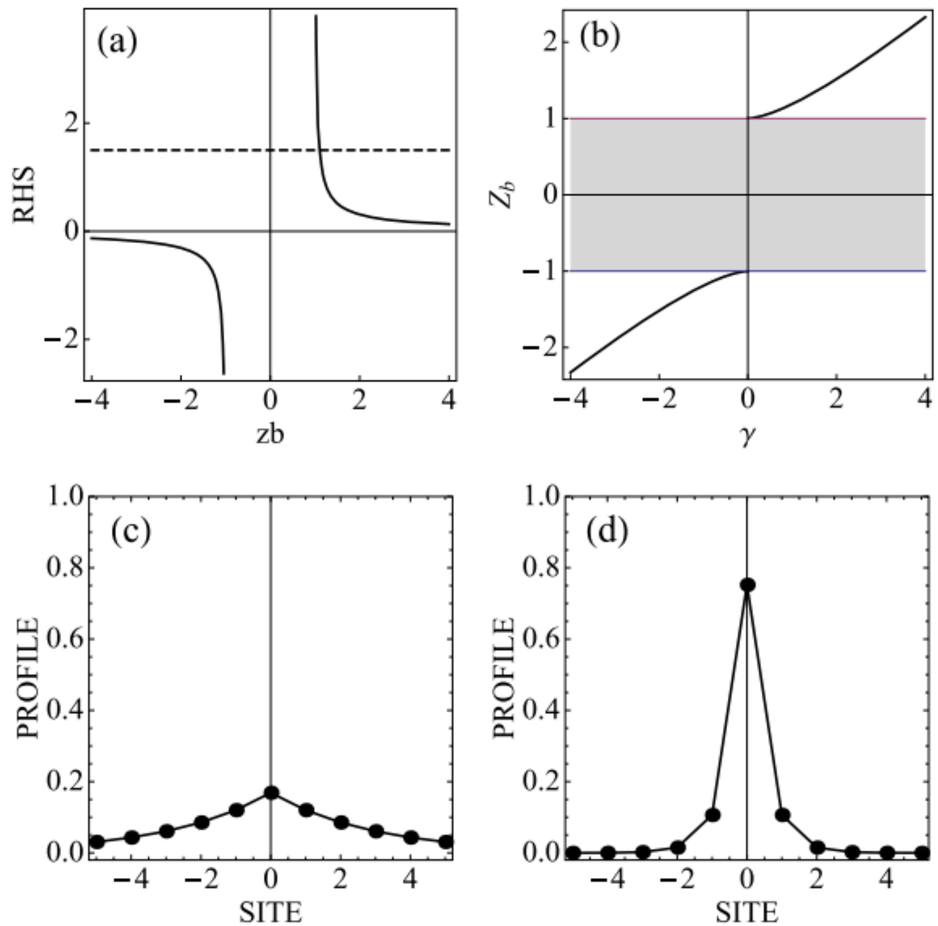
Bound state energy !

where

$$D(\gamma) = -1 + 39\gamma^2 + 15\gamma^4 + \gamma^6 + 6\sqrt{3}\gamma\sqrt{-1+11\gamma^2+\gamma^4}.$$

$$\left|E_n^{(b)}\right|^2 = \frac{\sqrt{z_b^2 - 1}}{|z_b|}\left(z_b - \mathrm{sgn}(z_b)\sqrt{z_b^2 - 1}\right)^{2|n-d|}$$

Bound state profile !



Bulk impurity

Surface impurity (d = 0)

Have to take into account the presence of boundary at d=0

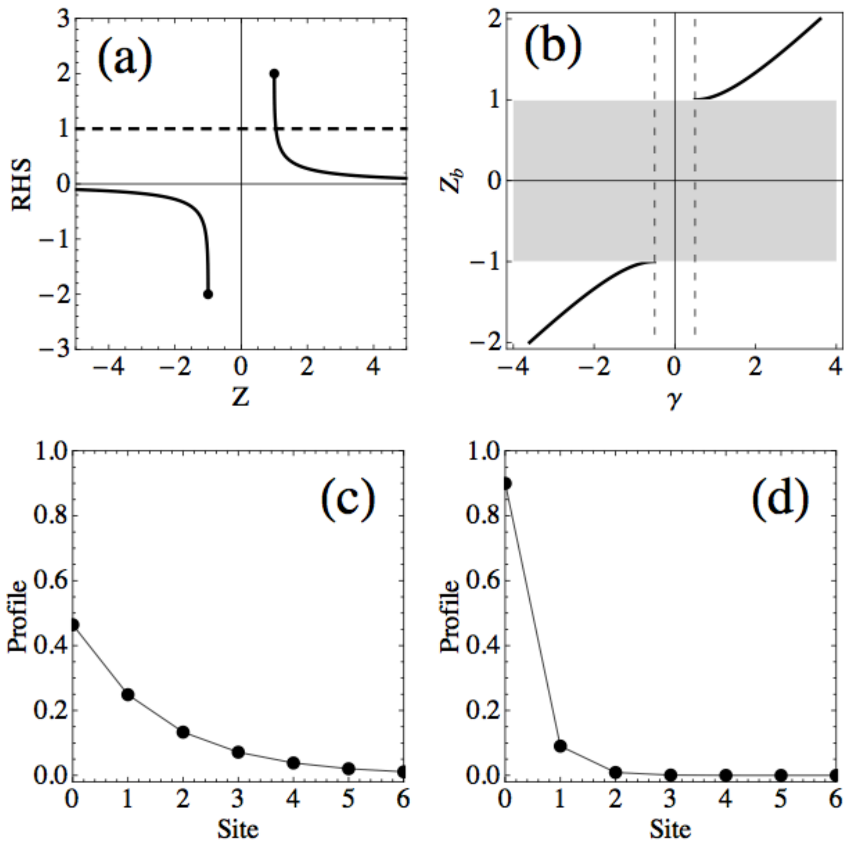
$$G_{mn}^{(0)} = G_{mn}^{\infty} - G_{m,-n-2}^{\infty}$$

$$G_{mn}^{(0)} = \frac{\mathrm{sgn(z)}}{\sqrt{z^2-1}}\left[z - \mathrm{sgn(z)}\sqrt{z^2-1}\right]^{|n-m|} - \frac{\mathrm{sgn(z)}}{\sqrt{z^2-1}}\left[z - \mathrm{sgn(z)}\sqrt{z^2-1}\right]^{|n+2+m|}$$

$$\frac{1}{\gamma} = \frac{2}{z + 3\ \mathrm{sgn(z)}\sqrt{z^2-1}}$$

$$z_b = (1/4)(-\gamma + 3\ \mathrm{sgn}(\gamma)\sqrt{2+\gamma^2})$$

$$|E_n^{(b)}|^2 = \alpha(z_b)(\ q(z_b)^{|n|} - q(z_b)^{|n+2|}\ )$$



Minimum nonlinearity strength needed

Surface impurity

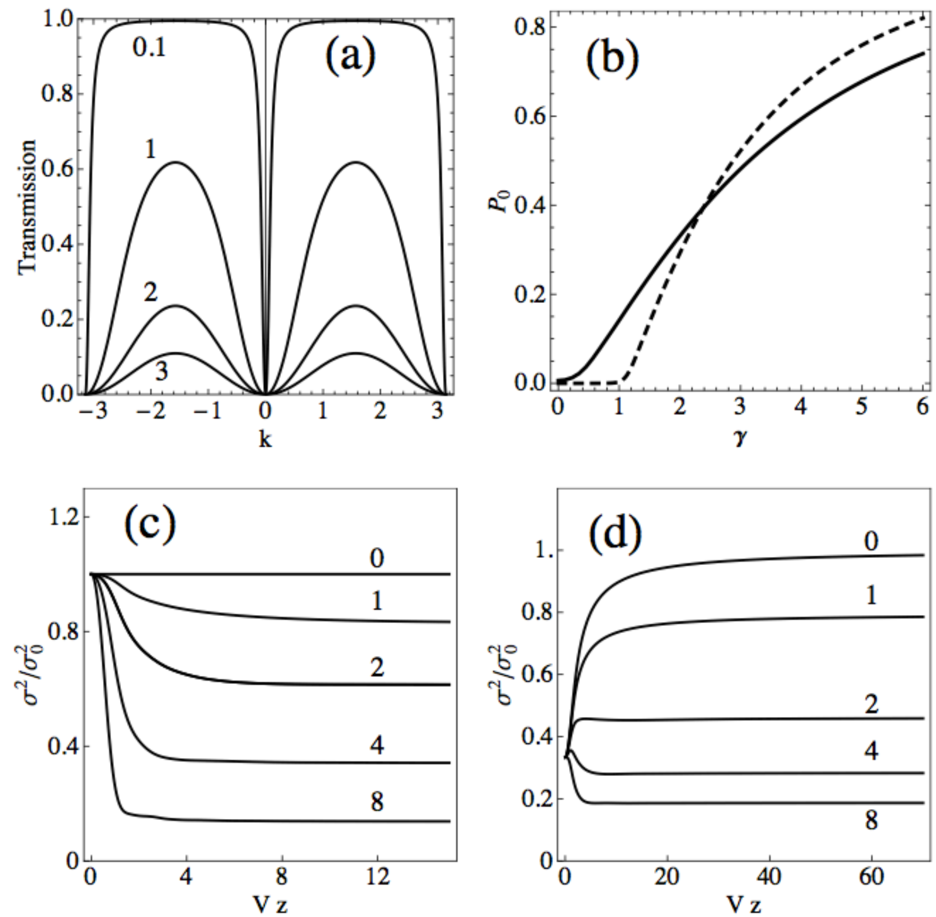
Dynamical properties

$$P_d = \lim_{T\to\infty} (1/L) \int_0^L |E_d(z)|^2 dz$$

$$\langle n^2 \rangle = \frac{\sum_n (n-d)^2 |E_n(z)|^2}{\sum_n |E_n(z)|^2}$$

ballistic propagation

$$\sigma(t) \to \sqrt{2}(Vt)$$



bulk surface

CONCLUSIONS

- Obtained Green function in closed form for 1D lattice with single saturable impurity.
- Used Green function to obtain energy and bound state profile in closed form.
- In bulk case an impurity state is always possible.
- For surface case a minimum nonlinearity is needed.
- Bulk case shows no selftrapping transition.
- Surface case shows selftrapping transition.
- Asymptotic propagation of optical power shows ballistic character