

Discrete Photonics in Waveguide Arrays

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<http://fisica.ciencias.uchile.cl/nonopt/NLOG.html>
<http://www.cefop.cl//>





Discrete photonics



Why study physics of discrete systems?

Testbed to test general phenomenology

Richer physics than continuous counterpart

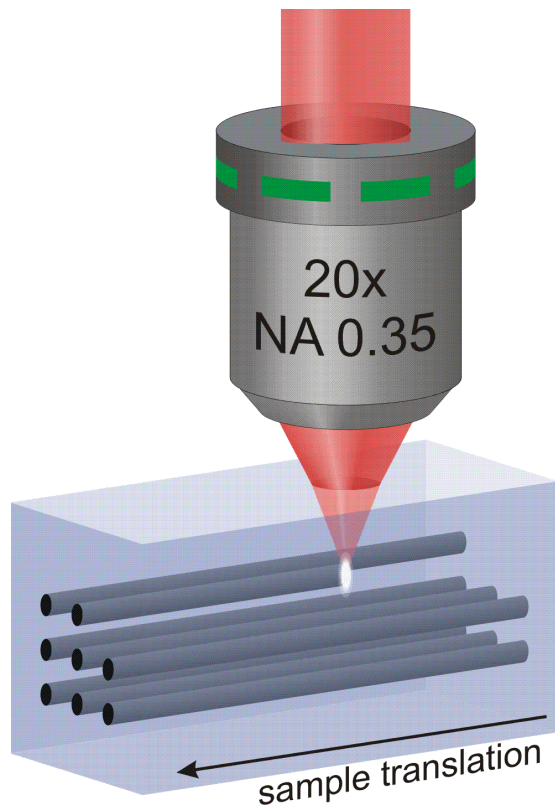
Greater potential for applications



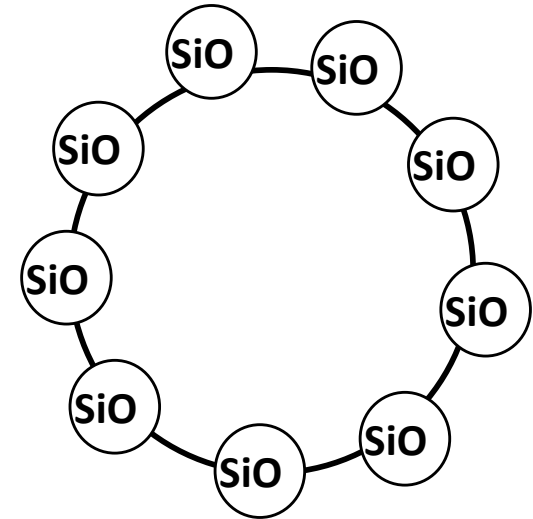
Waveguides in fused silica



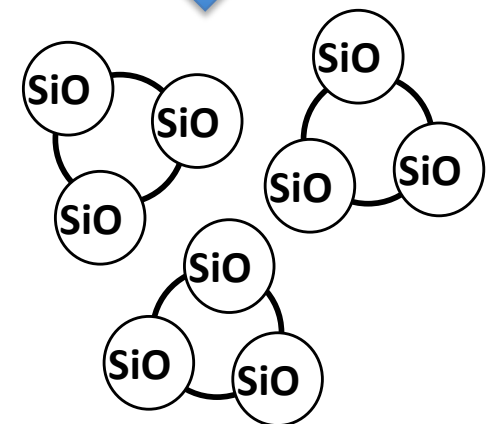
A. Szameit et al, Opt. Express 13,10552 (2005).



9-ring structure

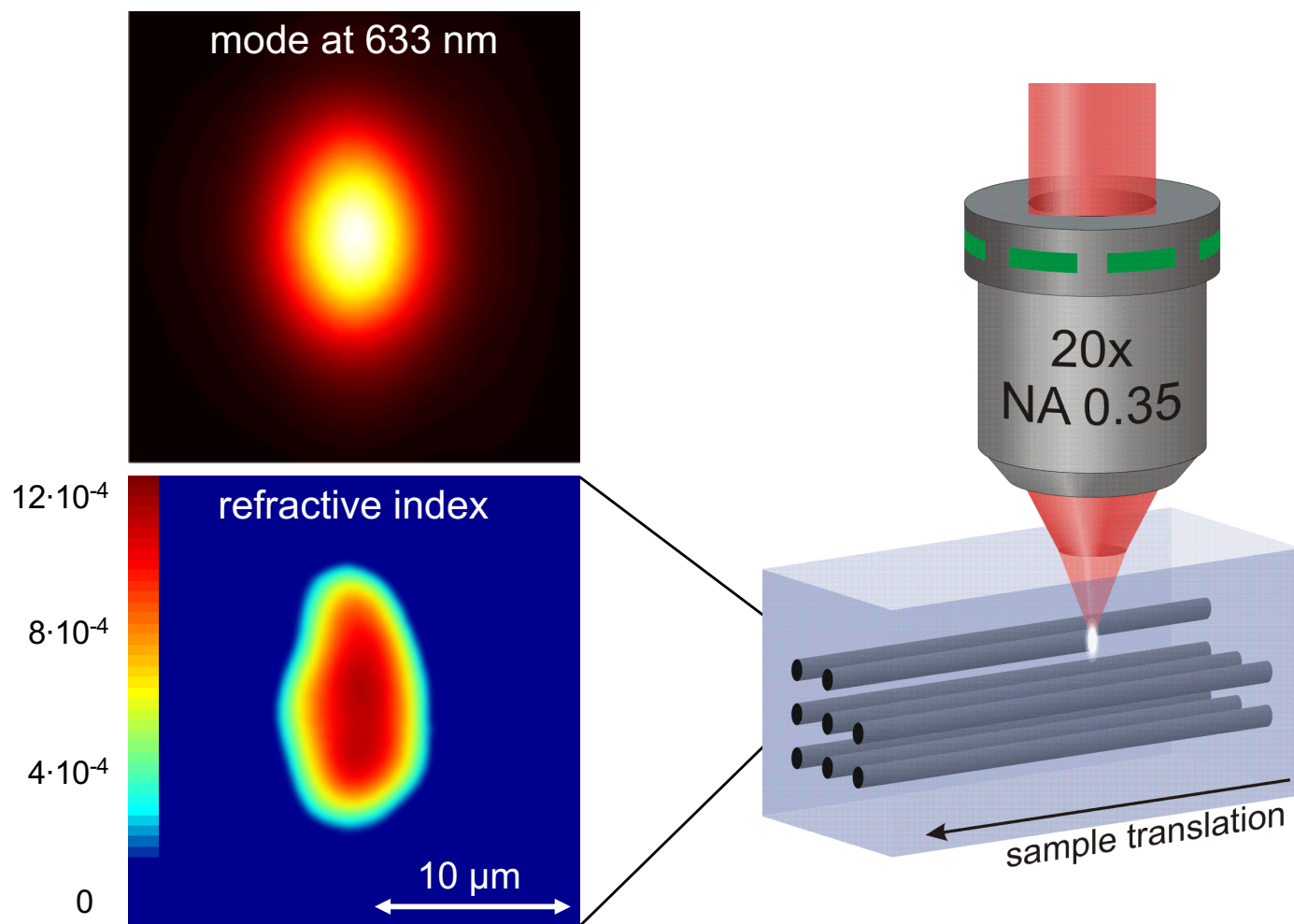


3-ring structures
(densification,
refractive index increase)





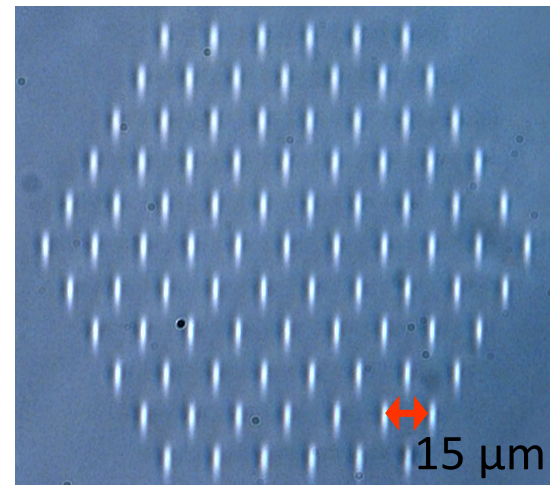
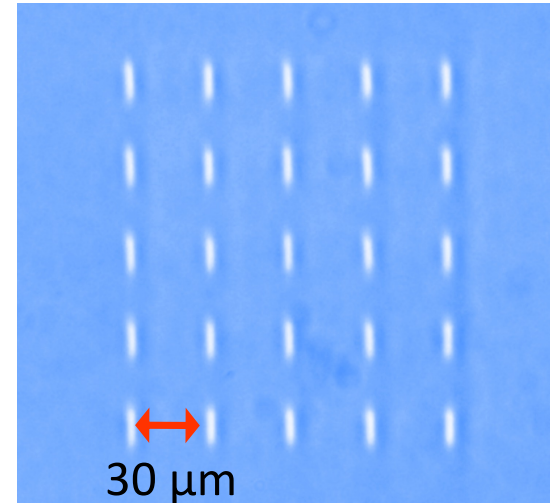
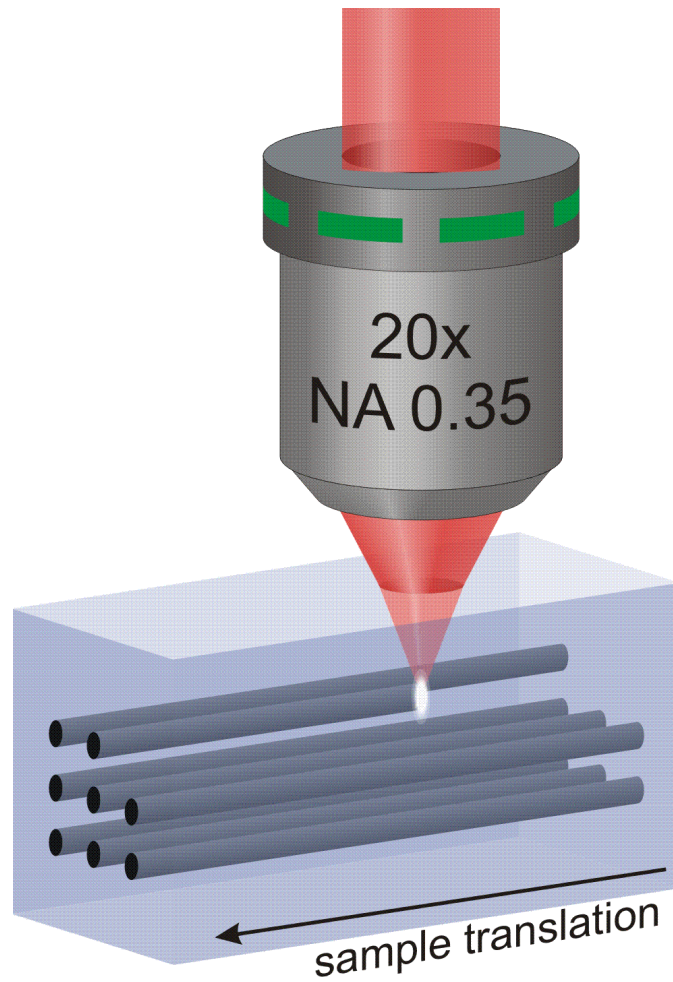
Waveguides in fused silica



typical $\Delta n = 10^{-3}$



Waveguides in fused silica



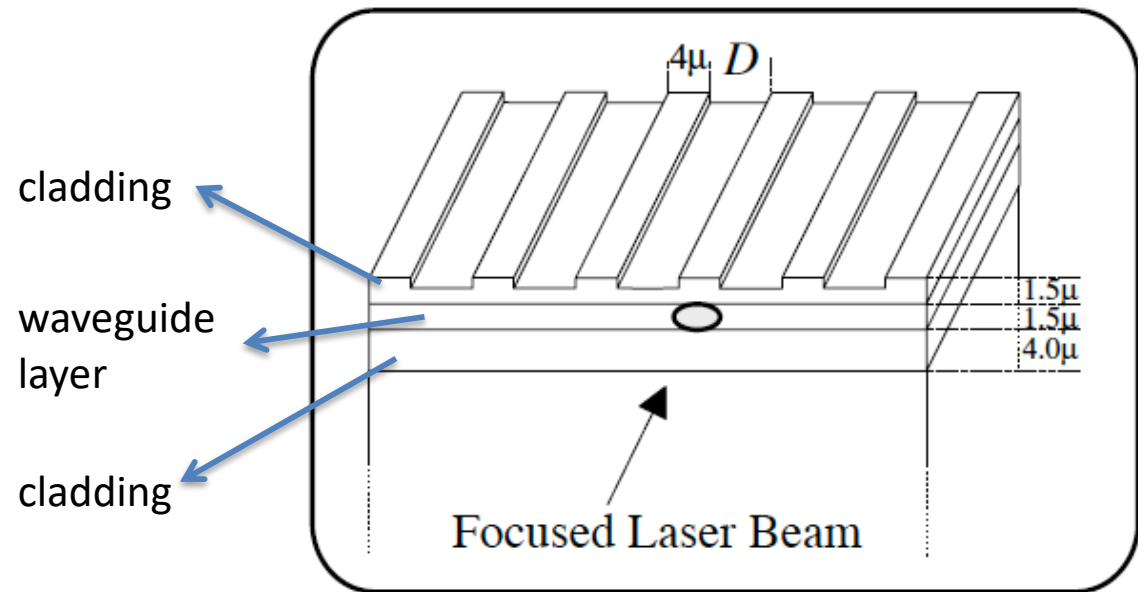
AS et al., Opt. Lett. **33**, 663 (2008).
AS et al., Appl. Phys. B **82**, 507 (2006).



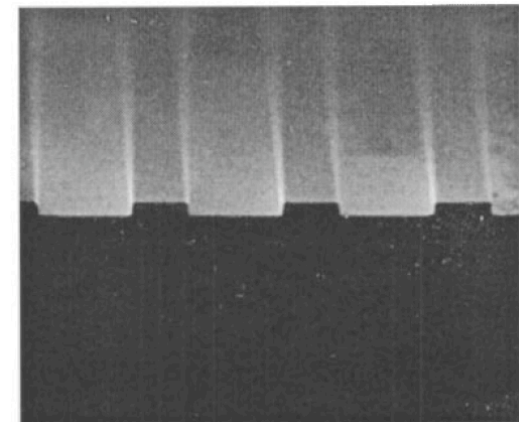
Semiconductor Waveguides



P. Millar, J.S. Aitchison, J.U. Kang, G.I. Stegeman, J. Opt. Soc. Am. B 14, 3224 (1997).

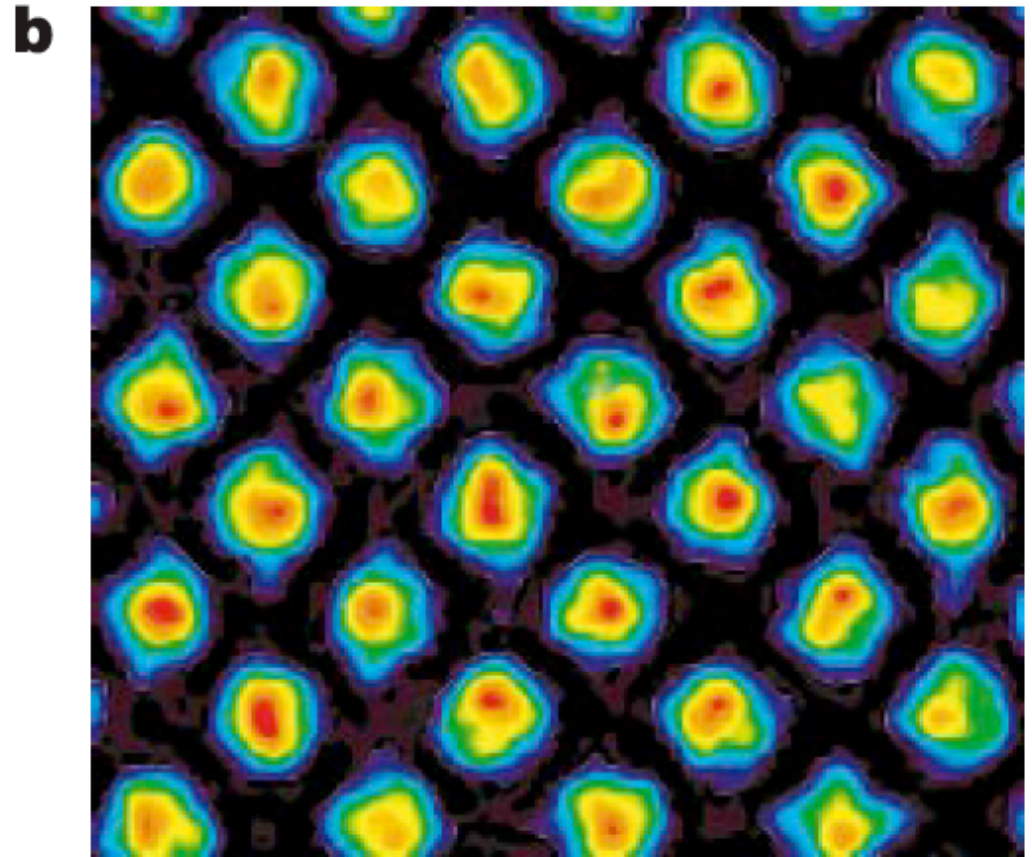
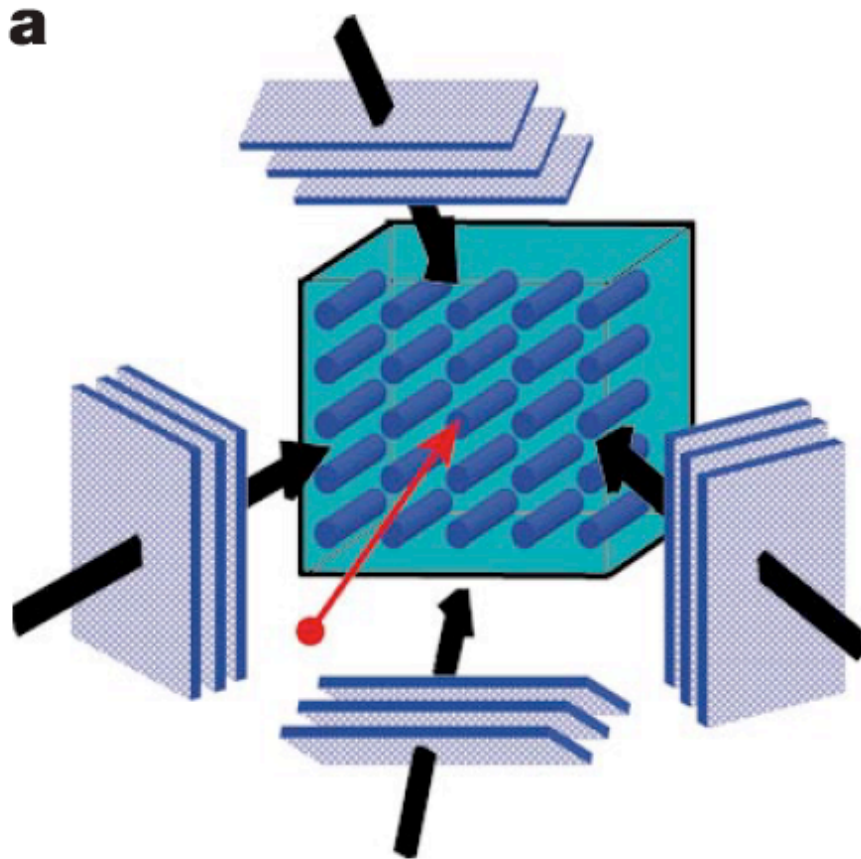


Substrate: Ga As
Cladding: $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$
Waveguide layer: $\text{Al}_{0.18}\text{Ga}_{0.82}\text{As}$





Photorefractive Waveguides



Light → releases electrons → drift → local E fields → electro-optic effect → distribution of refractive indices



Coupled-modes theory



Maxwell:

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial}{\partial t} \vec{D}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

$$\vec{D} = \vec{E} + \vec{P}$$

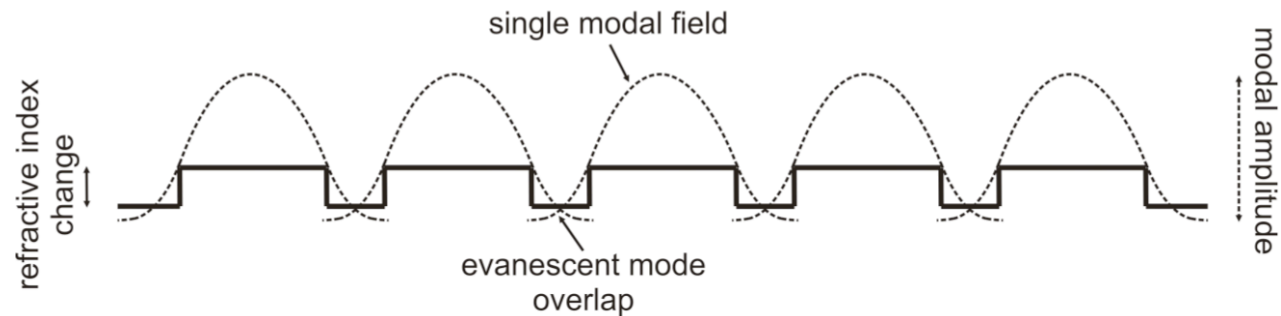
$$\vec{H} = \vec{B} + \vec{M}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2}$$

$$\vec{P} = \chi^{(1)} \vec{E} + \chi^{(3)} |\vec{E}|^2 \vec{E}$$



Teoría de modos acoplados



$$E(x, z) = \sum_{n=-\infty}^{\infty} C_n(z) \phi(x - x_n)$$

$$\left| \frac{d^2 C_n}{dz^2} \right| \ll k_0 \left| \frac{dC_n}{dz} \right| \quad n = n_0 + n_2 |E|^2 \quad \text{Kerr}$$

$$i \frac{dC_n}{dz} + V(C_{n+1} + C_{n-1}) + \gamma |C_n|^2 C_n = 0$$

Discrete nonlinear Schrodinger (DNLS) equation



Coupled-modes theory



$$P = \sum_n |C_n|^2$$

$$H = \sum_n \{V(C_n C_{n+1}^* + C_n^* C_{n+1}) + (\gamma/2)|C_n|^4\}$$

Conserved
quantities

$$q_n = C_n; \quad p_n = i C_n^*$$

$$(d/dt)q_n = \partial H / \partial p_n$$

$$(d/dt)p_n = -\partial H / \partial q_n$$

Hamiltonian
system

$$C_n = u_n \exp(i\beta z) \quad \text{Stationary mode}$$

$$-\beta u_n + (u_{n+1} + u_{n-1}) + \chi |u_n|^2 u_n = 0$$

Nonlinear eigenvalue equation



Coupled-modes theory



Finding the localized nonlinear mode

$$-EC_n + V(C_{n+1} + C_{n-1}) + \chi|C_n|^2 C_n = 0$$

$$\lambda \equiv E/V, \quad \phi_n \equiv \sqrt{\chi/V} C_n$$

$$-\lambda\phi_n + (\phi_{n+1} + \phi_{n-1}) + |\phi_n|^2 \phi_n = 0$$

$$\vec{F}(\vec{\phi}) = 0 \quad \text{use Newton-Raphson}$$

Need good seed (anticontinuous limit)

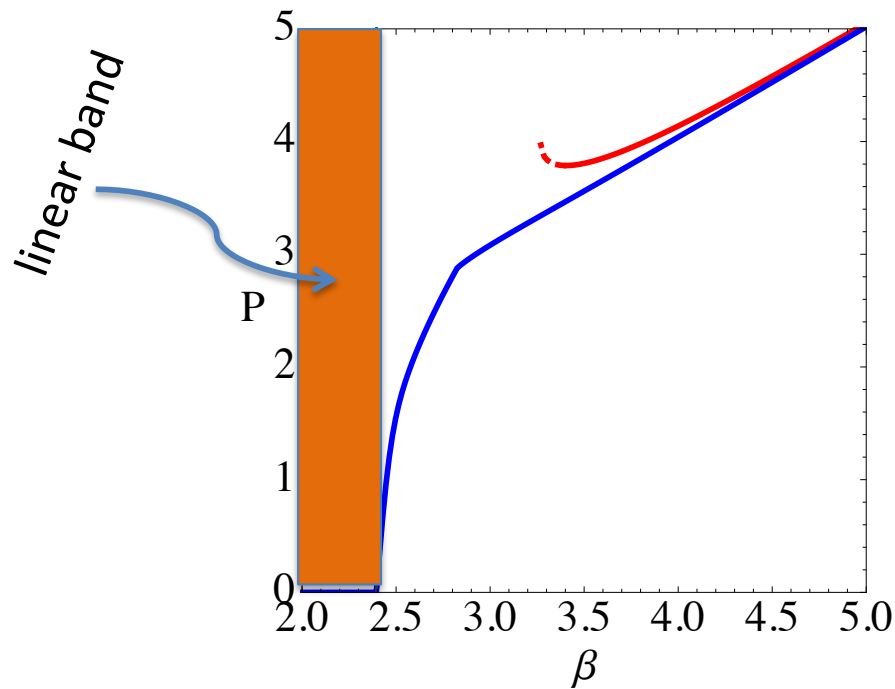
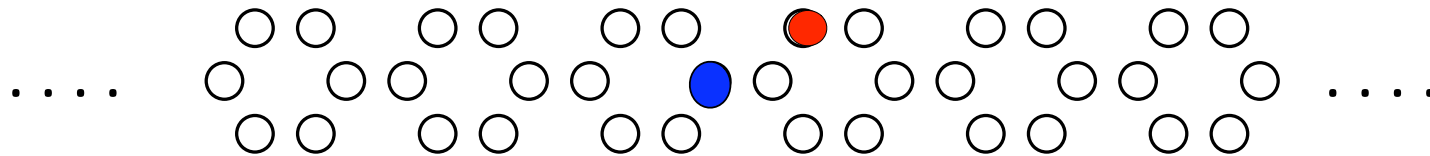
Find many solution families (characterized by power
vs prop.const. curve)



Coupled-modes theory



Example: Graphene ribbon



$$P = \sum_n |\phi_n|^2$$

Conserved quantity



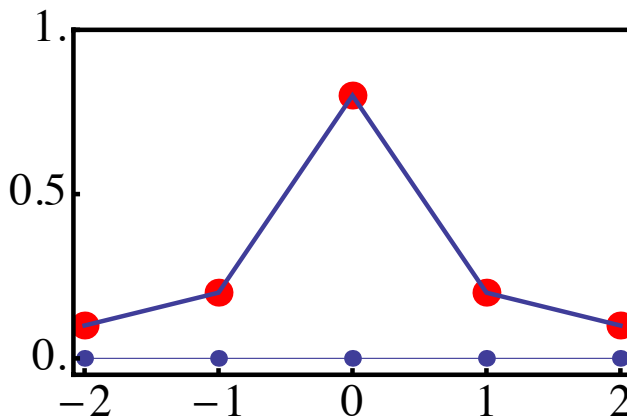
Coupled-modes theory



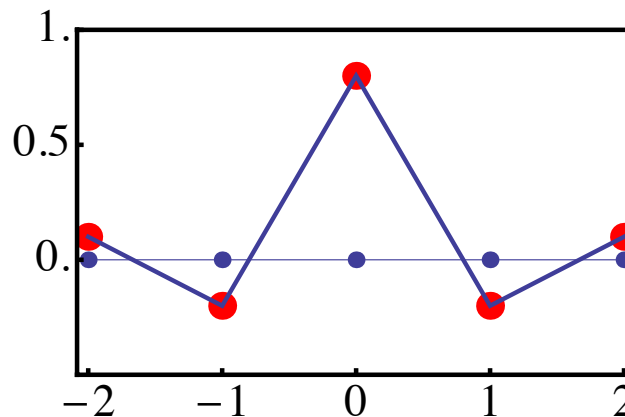
Pure 1D CASE



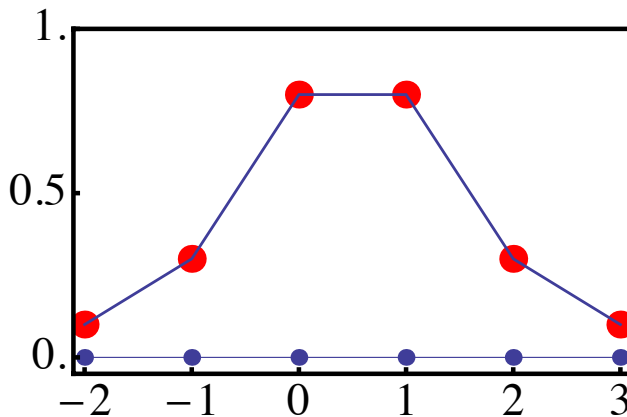
odd
unstaggered



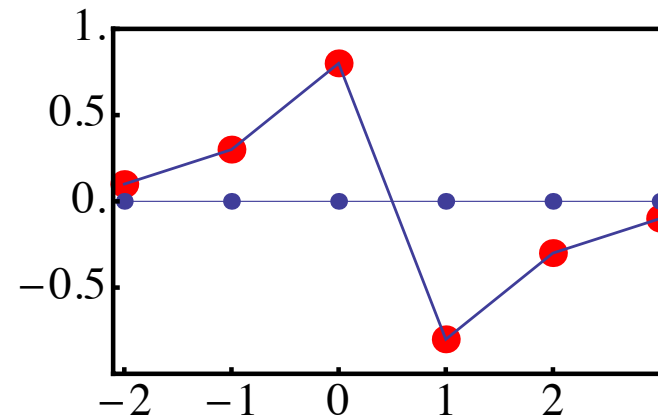
odd
staggered



even



twisted





Coupled-modes theory



Linear stability

$$C_n(z) = \phi_n e^{-i\lambda z} \quad \text{sol. of DNLS}$$

$$C_n(z) \rightarrow (\phi_n + \delta\phi_n) e^{-i\lambda z}, \quad |\delta\phi_n/\phi_n| \ll 1$$

$$\implies \text{Equation for } \delta\phi_n = \delta u_n + i\delta v_n$$

$$\text{define } \delta\vec{u} = (\delta u_1, \delta u_2, \dots, \delta u_N), \quad \delta\vec{v} = (\delta v_1, \delta v_2, \dots, \delta v_N)$$

$$\mathcal{A}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + \phi_n^2) \delta_{n,m}$$

$$\mathcal{B}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + 3\phi_n^2) \delta_{n,m}$$

$$\boxed{\delta\ddot{\vec{U}} + \mathcal{B}\mathcal{A} \delta\vec{U} = 0 \quad \text{and} \quad \delta\ddot{\vec{V}} + \mathcal{A}\mathcal{B} \delta\vec{V} = 0}$$



Coupled-modes theory



$\{m\}$ =eigenvalues of $\mathcal{AB}$ = eigenvalues of $\mathcal{BA}$

instability gain

$$G^* = \text{Max} \left\{ \sqrt{(1/2)(-\text{Re}[m] + \sqrt{\text{Re}[m]^2 + \text{Im}[m]^2})} \right\}$$

$$G^* = 0 \quad \text{stable}$$

$$G^* > 0 \quad \text{unstable}$$

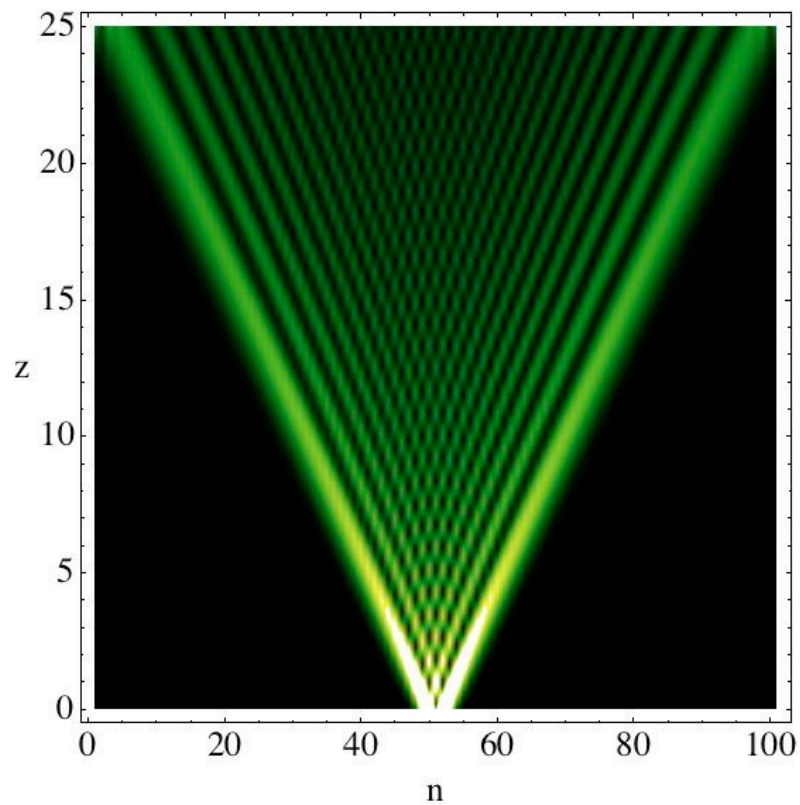


Coupled-modes theory

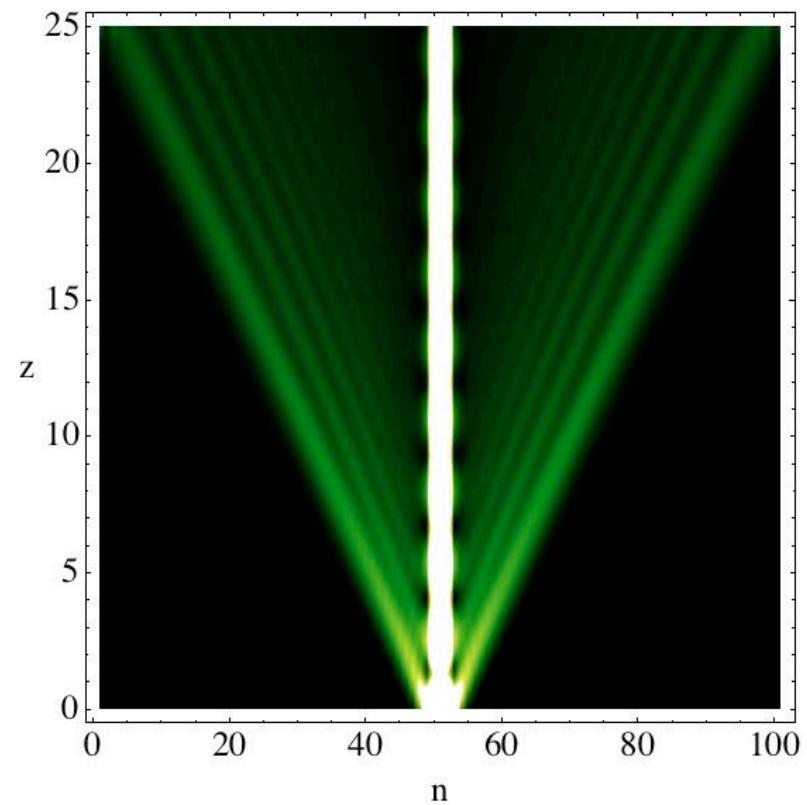


Numerical propagation

Discrete diffraction

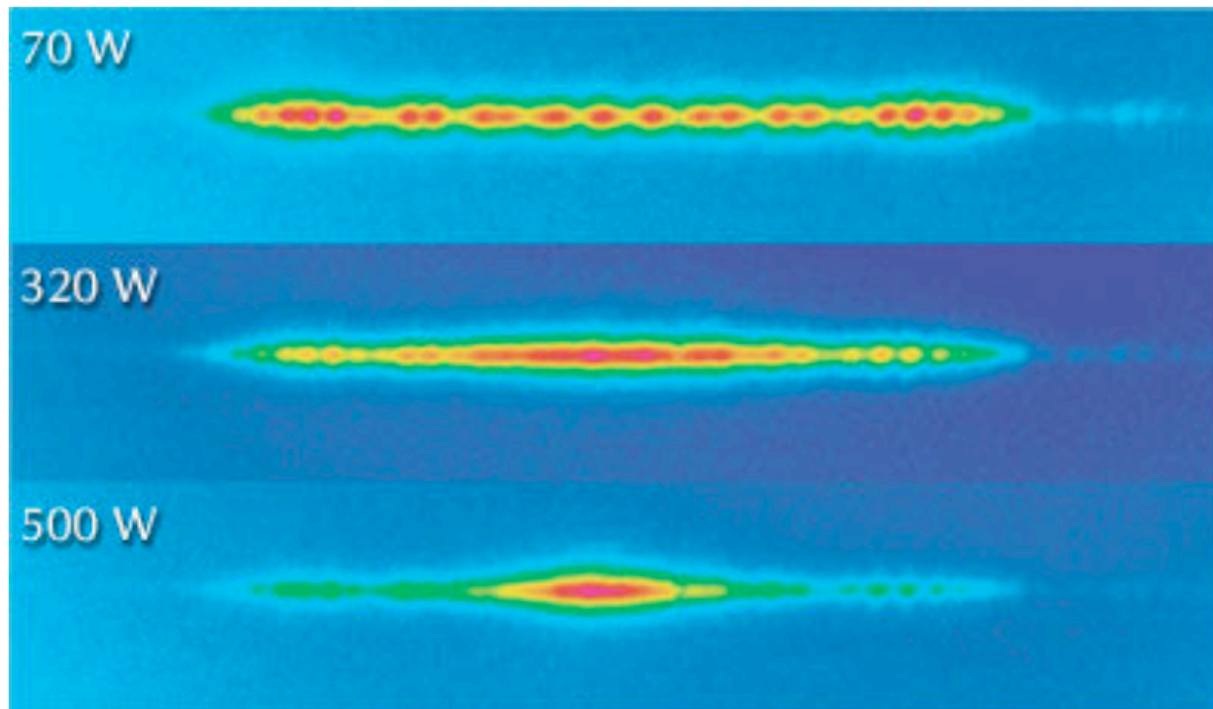


Discrete soliton formation





First experimental observation of discrete soliton



Difraccion discreta

Soliton discreto

H. Eisenberg et al, PRL 81, 3383 (1998).