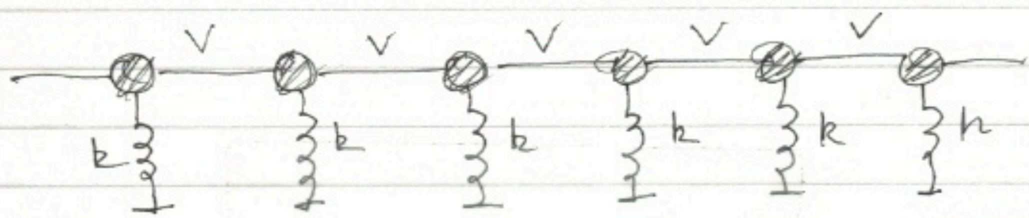
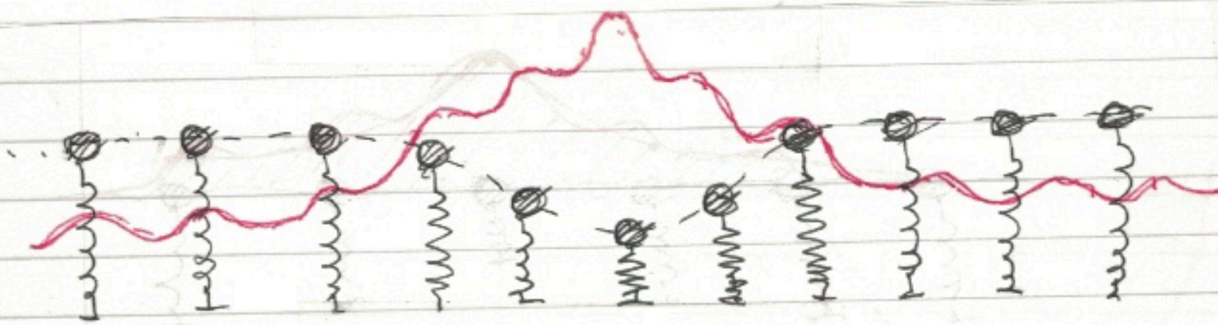
20 Sept. 2008

# Sistemas Deformable



Red tight-binding + Osciladores anarmónicos tipo Einstein

$$i\frac{dC_n}{dt} = V(C_{n+1}+C_{n-1}) + \alpha u_n C_n \qquad (1)$$

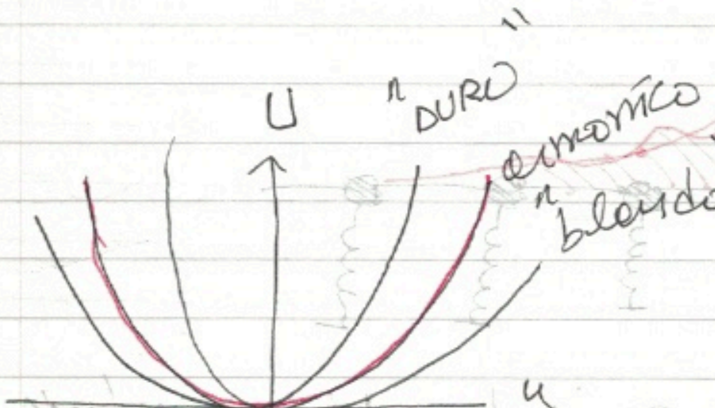
$$m\frac{d^2u_n}{dt^2} = -\left(\frac{\partial U}{\partial u_n}\right) - \alpha |C_n|^2 \qquad (2)$$

$m \to 0$ (límite antiadiabático): $\left(\frac{\partial U}{\partial u_n}\right) = -\alpha|C_n|^2$ (2')

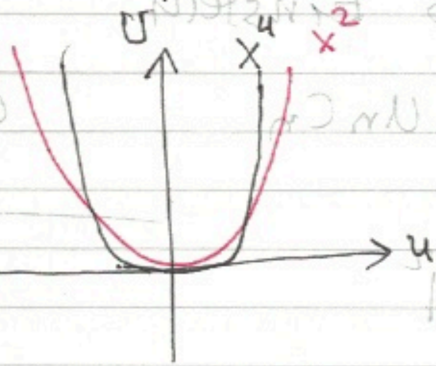
$$\Rightarrow u_n(t) = u_n(\alpha|C_n(t)|^2) \qquad (3)$$

$$\Rightarrow i\frac{dC_n}{dt} = V(C_{n+1}+C_{n-1}) + \alpha u_n(\alpha|C_n|^2)C_n \qquad (4)$$

Caso "standard": $U(x) = \frac{1}{2}Kx^2 \rightarrow$ Ec. DNLS con $\chi \equiv \alpha^2/K$



U
"DURO"
"ARMONICO"
"BLANDO"
u

U
$X^4$
$X^2$
u

Conservation of total probability

$$i\frac{dc_n}{dt} = v(c_{n+1}+c_{n-1}) + g(|c_n|^2)c_n \qquad / \, c_n^*$$

$$i\dot{c}_n c_n^* = v\, c_{n+1}c_n^* + v\, c_{n-1}c_n^* + g(|c_n|^2)|c_n|^2 \qquad (1)$$

$$-i\dot{c}_n^* = v(c_{n+1}^* + c_{n-1}^*) + g(|c_n|^2)c_n^* \qquad / \, c_n$$

$$\Rightarrow -i\dot{c}_n^* c_n = v(c_{n+1}^* c_n) + v\, c_{n-1}^* c_n + g(|c_n|^2)|c_n|^2 \qquad (2)$$

$$(1)-(2): \quad i\frac{d}{dt}|c_n|^2 = v\, c_{n+1}c_n^* + v\, c_{n-1}c_n^* - v\, c_{n+1}^* c_n - v\, c_{n-1}^* c_n$$

$$\Rightarrow i\frac{d}{dt}\sum_n |c_n|^2 = v\sum_{\substack{n \\ \to n-1}} c_{n+1}c_n^* - v\sum_n c_{n-1}^* c_n + v\sum_n c_{n-1}c_n^* - v\sum_{n\to n-1} c_{n+1}^* c_n$$

$$= \underbrace{v\sum_n c_n c_{n-1}^* - v\sum_n c_{n-1}^* c_n}_{0} + \underbrace{v\sum_n c_{n-1}c_n^* - v\sum_n c_n^* c_{n-1}}_{0}$$

$$= 0 .$$

Ejercicio: Demostrar lo mismo para una cadena semi-infinita

(i) Potencial débilmente anarmónico

$$U(u) = \frac{1}{2} K u^2 + \frac{1}{4} K_3 u^4 \qquad (5)$$

$$(2) \Rightarrow \quad K u_n + K_3 u_n^3 = -\alpha |C_n|^2$$

A orden cero: $\quad u_n^{(0)} \simeq -(\alpha/K)|C_n|^2$

$$u_n = u_n^{(0)} + \delta u_n \quad , \text{ con } \delta u_n \ll |u_n^{(0)}|$$

$$u_n^3 = (u_n^{(0)} + \delta u_n)^3 = u_n^{(0)3} + 3 u_n^{(0)2} \delta u_n + O(\delta u_n)^2$$

$$\Rightarrow K(u_n^{(0)} + \delta u_n) + K_3 (u_n^{(0)3} + 3 u_n^{(0)2} \delta u_n) = -\alpha |C_n|^2$$

$$\underline{K u_n^{(0)}} + K \delta u_n + K_3 u_n^{(0)3} + 3 K_3 u_n^{(0)2} \delta u_n = \underline{-\alpha |C_n|^2}$$

$$\Rightarrow (K + 3 K_3 u_n^{(0)2}) \delta u_n = -K_3 u_n^{(0)3}$$

$$\Rightarrow \delta u_n = \frac{-K_3 u_n^{(0)3}}{K + 3K_3 u_n^{(0)2}} = \frac{-(K_3/K) u_n^{(0)3}}{1 + 3(K_3/K) u_n^{(0)2}} \simeq -(K_3/K) u_n^{(0)3}$$

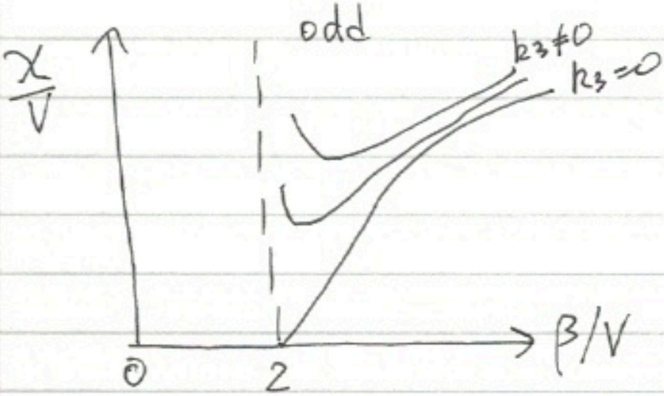
$$\therefore u_n \simeq u_n^{(0)} - (K_3/K) u_n^{(0)3} = u_n^{(0)} \left[1 - \frac{K_3}{K} u_n^{(0)2}\right] = -\left(\frac{\alpha}{K}\right)|C_n|^2 \left\{1 - \frac{K_3}{K}\left(\frac{\alpha}{K}\right)^2 |C$$

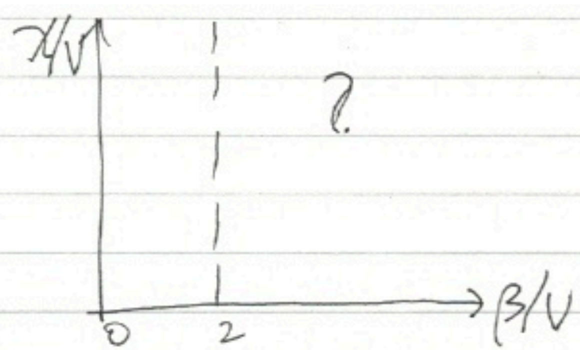
$$\Rightarrow \boxed{i \frac{dC_n}{dt} = V(C_{n+1} + C_{n-1}) - \chi\left(1 - \frac{K_3}{K^2} \chi |C_n|^4\right) C_n .} \qquad (6)$$

Modos localizados: $C_n(t) = C_n e^{-i\beta t}$

$$\beta C_n = V(C_{n+1} + C_{n-1}) - \chi\left[1 - \left(\frac{k_3}{k_2}\right)\chi|C_n|^4\right]C_n$$

Fijar $k_3/k^2 \ll 1$ y hallar modos par e impar




¿Es posible estabilizar el modo par, para alguna elección "astuta" de $k_3$?

(ii) Potencial "DURO" $U(u) = k(\cosh(u)-1)$

se aproxima a $\frac{1}{2}ku^2$ cuando $u \ll 1$.

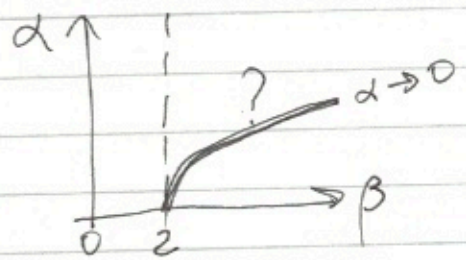
$$\frac{dU}{du_n} = k\sinh(u_n) = -\alpha|C_n|^2 \Rightarrow u_n = -\sinh^{-1}\left(\frac{\alpha}{k}|C_n|^2\right)$$

$$\Rightarrow i\frac{dC_n}{dt} = V(C_{n+1}+C_{n-1}) - \alpha\sinh^{-1}\left(\frac{\alpha}{k}|C_n|^2\right)C_n.$$ Ec. dinámica

Estados estacionarios: $C_n(t) = C_n e^{-i\beta t}$.

$$\beta C_n = V(C_{n+1}+C_{n-1}) - \alpha\sinh^{-1}\left(\frac{\alpha}{k}|C_n|^2\right)C_n.$$

Analizar la $\exists$ de modo par e impar y su estabilidad (tomar $V = 1 = k$)

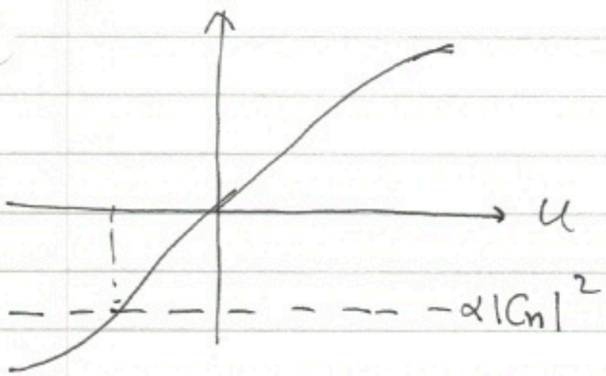


(iii) Potencial "blando" monótono/. creciente

$$U(u) = (K/3)[u^2+|u|-\log(1+|u|)]$$

$$U(u) \to \tfrac{1}{2} k u^2 \quad (u \to 0)$$

$$\frac{dU}{du} = (K/3)\left[2u+\frac{u}{|u|}-\frac{(u/|u|)}{(1+|u|)}\right] = \frac{k}{3}\left[2u+\mathrm{sgn}(u)\left(1-\frac{1}{1+|u|}\right)\right]$$




$\alpha<0 \Rightarrow u_n>0$

$\alpha>0 \Rightarrow u_n<0$

$$\alpha<0 \Rightarrow u_n = \frac{-3\alpha|C_n|^2-3k+\sqrt{9\alpha^2|C_n|^4-6\alpha|C_n|^2k+9k^2}}{4k}$$

$$\alpha>0 \Rightarrow u_n = -\left(\frac{3\alpha|C_n|^2-3k+\sqrt{9\alpha^2|C_n|^4+6\alpha|C_n|^2k+9k^2}}{4k}\right)$$

$$u_n = -\mathrm{sgn}(\alpha)\left(\frac{3|\alpha||C_n|^2-3k+\sqrt{9\alpha^2|C_n|^4+6|\alpha|k|C_n|^2+9k^2}}{4k}\right)$$

$$\Rightarrow i\frac{dC_n}{dt} = V(C_{n+1}+C_{n-1}) - |\alpha|\left\{\frac{3}{4}\left(\frac{|\alpha|}{k}\right)|C_n|^2 - \frac{3}{4} + \frac{1}{4}\sqrt{9\left(\frac{\alpha}{k}\right)^2|C_n|^4 + 6\frac{|\alpha|}{k}|C_n|^2 +}\right.$$

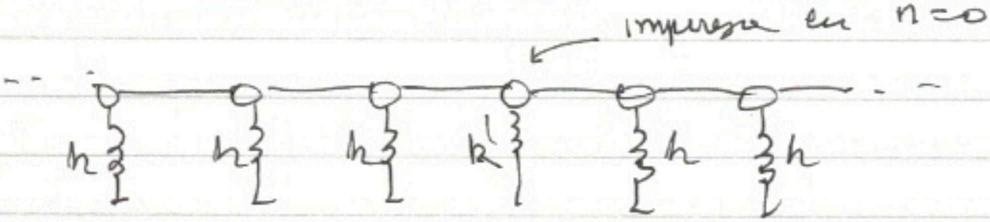
$$\times |C_n$$

$$C_n(t) = e^{-i\beta t} C_n$$

$$\boxed{\beta C_n = V(C_{n+1}+C_{n-1}) - F(\alpha, k, |C_n|^2)\, C_n}$$

Find localized nonlinear mode for $V = 1 = k$ as a fn. of $\beta, \alpha$. Constraint: $\sum_n |C_n|^2 = 1$

# Impureza vibracional




$$i\dot{C}_n = V(C_{n+1}+C_{n-1}) + \alpha u_n C_n$$

$$\underbrace{m\ddot{u}_n}_{0} = -\alpha|C_n|^2 + (\delta_{n0}(k-k')+k)u_n$$

$$u_n = \begin{cases} -(\alpha/k)|C_n|^2 & n \neq 0 \\ -(\alpha/k')|C_n|^2 & n = 0 \end{cases}$$

$$\Rightarrow \quad i\dot{C}_n = V(C_{n+1}+C_{n-1}) - (\alpha^2/k)|C_n|^2 C_n$$

$$i\dot{C}_0 = V(C_1+C_{-1}) - (\alpha^2/k')|C_0|^2 C_n$$

ó $$i\dot{C}_n = V(C_{n+1}+C_{n-1}) - (\chi + \delta_{n0}(\chi'-\chi))|C_n|^2 C_n$$

¿Cómo afecta la impureza vibracional a los estados del sist. homogéneo?

# Teoría de Modos Acoplados
(A.K.A. Tight-binding in Solid state theory).

Partimos de las ecs. de Maxwell para un medio sin fuente (o sea sin cargas ni corrientes)

$$\left.\begin{array}{l} \vec{\nabla}\times\vec{\mathcal{H}} = \dfrac{\partial \vec{D}}{\partial t} \quad ; \quad \vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\cdot\vec{D} = 0 \quad ; \quad \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{D} = \vec{D}(\vec{E}) \quad y \quad \vec{B} = \vec{B}(\vec{\mathcal{H}}) \end{array}\right\} \quad (1)$$

$\vec{B}$ → inducción magnética.

$\vec{D}$ → desplazamiento eléctrico; $\vec{E}$ → campo eléctrico; $\vec{\mathcal{H}}$ → campo magnético

Es costumbre definir $\vec{P}$ y $\vec{M}$ por medio de:

$$\left.\begin{array}{l} \vec{D} = \epsilon_0\vec{E} + \vec{P} \\ \vec{B} = \mu_0\vec{\mathcal{H}} + \mu_0\vec{M} \end{array}\right. \qquad \left.\begin{array}{l} \vec{P}: \text{densidad polarización} \\ \vec{M}: \text{〃 magnetización} \end{array}\right\} \quad (2)$$

en gral, $\vec{P} = \vec{P}(\vec{E})$ y $\vec{M} = \vec{M}(\vec{\mathcal{H}})$

Supondremos un medio NO magnético $\Rightarrow \boxed{\vec{M} = 0}$ (3)

$\Rightarrow \vec{B} = \mu_0\vec{\mathcal{H}}$. (4)

A. Medio lineal, no-dispersivo, homogéneo e isotrópico

$$\vec{P} \propto \vec{\mathcal{E}} \qquad \vec{P}(t) \propto \vec{\mathcal{E}}(t)$$

$$\Rightarrow \boxed{\vec{P} = \epsilon_0 \chi \vec{\mathcal{E}}} \quad (5)$$ $\chi$ : susceptibilidad eléctrica

$$(3) \Rightarrow \vec{D} = \epsilon_0 \vec{\mathcal{E}} + \epsilon_0 \chi \vec{\mathcal{E}} = \epsilon_0 (1+\chi) \vec{\mathcal{E}} = \epsilon \vec{\mathcal{E}} \quad (6)$$

$\frac{\epsilon}{\epsilon_0}$ se llama la cte. dieléctrica del medio.

con todo esto, tenemos $\vec{\nabla} \cdot \vec{\mathcal{E}} = 0 = \vec{\nabla} \cdot \vec{\mathcal{H}}$

$$\vec{\nabla} \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t} \quad ; \quad \vec{\nabla} \times \vec{\mathcal{E}} = -\mu_0 \frac{\partial \vec{\mathcal{H}}}{\partial t} \quad (7)$$

$\Rightarrow$ cada comp. de $\vec{\mathcal{E}}$ y $\vec{\mathcal{H}}$ satisface

$$\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{con} \quad c = \frac{c_0}{n}$$

$c_0 \equiv 1/\sqrt{\mu_0 \epsilon_0}$, $n \equiv \sqrt{\epsilon/\epsilon_0} = \sqrt{1+\chi}$ "índice de refracción"

B. Medios no-lineal, dispersivo, inhomogéneo o anisotrópico

Inhomogéneo + lineal + no-dispersivo + isotrópico

$\rightarrow \chi = \chi(\vec{r})$ y $\epsilon = \epsilon(\vec{r})$

$\rightarrow \nabla^2 \mathcal{E} - \frac{1}{c^2} \frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$ ; $c(\vec{r}) = c_0/n(\vec{r})$ , $n(\vec{r}) = \sqrt{\epsilon(\vec{r})/\epsilon_0}$

$\nabla \times \mathcal{E} = -\frac{\partial}{\partial t} \ \ (\nabla \times$

$$\nabla \times (\nabla \times \mathcal{E}) = \nabla(\nabla \cdot \mathcal{E}) - \nabla^2 \mathcal{E} = -\mu_0 \frac{\partial^2 D}{\partial t^2}$$

y $\vec{\nabla} \cdot (\epsilon \vec{\mathcal{E}}) = 0 = \epsilon \vec{\nabla} \cdot \vec{\mathcal{E}} + \vec{\mathcal{E}} \cdot \vec{\nabla} \epsilon$

$$\Rightarrow \vec{\nabla} \cdot \vec{\mathcal{E}} = -\frac{1}{\epsilon} \vec{\nabla} \epsilon \cdot \vec{\mathcal{E}} = -\vec{\nabla}(\ln \epsilon) \cdot \vec{\mathcal{E}} \simeq 0 \qquad (8)$$

tensor de susceptibilidad

medio anisotrópico: $P_i = \epsilon_0 \chi_{ij} \mathcal{E}_j$
lineal

tensor de permitividad eléctrica

$$\longrightarrow D_i = \epsilon_{ij} \mathcal{E}_j \qquad (9)$$

medio dispersivo:

$$\vec{P}(t) = \epsilon_0 \int_{-\infty}^{\infty} \chi(t-t') \vec{\mathcal{E}}(t') dt'$$

$\hookrightarrow$ fn. impulso-respuesta (10)

$\Rightarrow P(\omega) = \chi(\omega) \mathcal{E}(\omega)$

medio no-lineal (pero homogéneo, isotrópico y no-dispersivo

de $\underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{\mathcal{E}})}_{\vec{\nabla}(\vec{\nabla} \cdot \vec{\mathcal{E}}) - \vec{\nabla}^2 \vec{\mathcal{E}}} = -\mu_0 \partial^2 \vec{D} / \partial t^2$ , $\vec{D} = \epsilon_0 \vec{\mathcal{E}} + \vec{P}(\vec{\mathcal{E}})$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{\mathcal{E}}) - \vec{\nabla}^2 \vec{\mathcal{E}} = -\epsilon_0 \mu_0 \frac{\partial^2 \mathcal{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

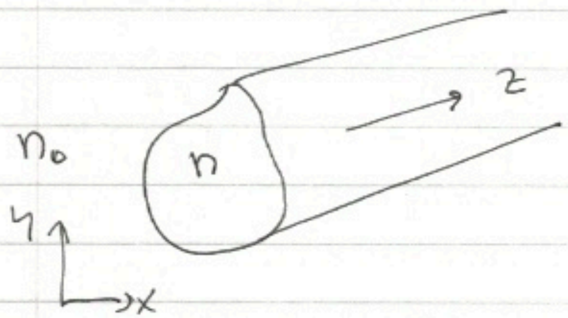
y como $\vec{D} = \epsilon \vec{\mathcal{E}} \Rightarrow 0 = \vec{\nabla} \cdot \vec{D} \Rightarrow 0 = \vec{\nabla} \cdot \vec{\mathcal{E}}$

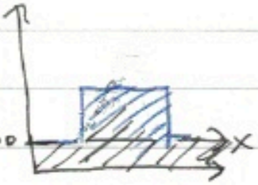
$$\rightarrow \boxed{\nabla^2 \mathcal{E} - \frac{1}{c_0^2} \frac{\partial^2 \mathcal{E}}{\partial t^2} = \mu_0 \frac{\partial^2 P(\mathcal{E})}{\partial t^2}} \qquad (11)$$

para medio no-dispersivo, $\vec{P} = \Psi(\vec{E})$

$$\Rightarrow \nabla^2 E - \frac{1}{c_0^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 \Psi(E)}{\partial t^2}.$$

## GUIA de ondas : 1 GUIA



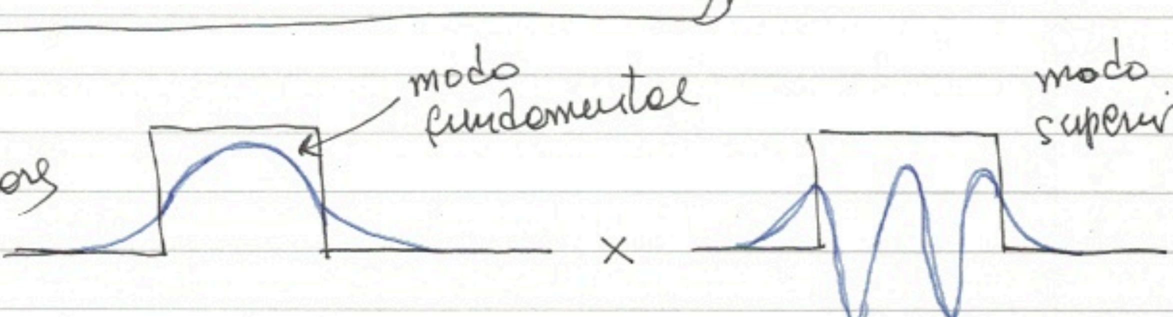

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0 \qquad c = \frac{c_0}{n} = \begin{cases} c_0/n_0 & \text{fuera} \\ c_0/n & \text{dentro} \end{cases}$$

$$u(\vec{r},t) = e^{i\omega t}\, u(\vec{r}) \Rightarrow \nabla^2 u(\vec{r}) + \left(\frac{\omega}{c}\right)^2 u(\vec{r}) = 0$$

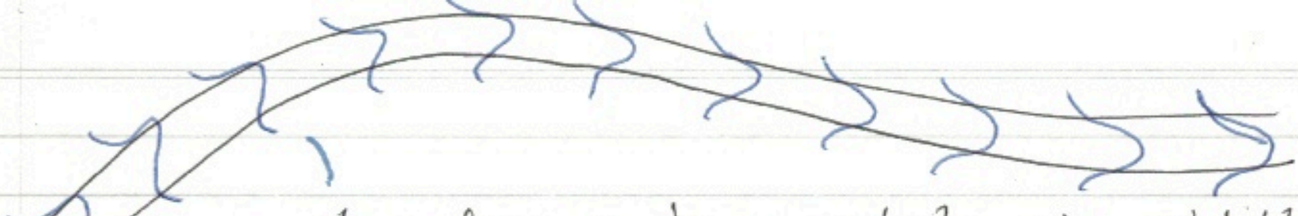
Ponemos $u(\vec{r}) = e^{i\beta z} u(x,y)$

$$\Rightarrow \boxed{\nabla_\perp^2 u(x,y) - \left(\beta^2 - \left(\frac{\omega}{c}\right)^2\right) u(x,y) = 0}$$ Ec. de autovalores

ejemplo:
soluc. para valores
específicos de
$\beta \to$ modos
permitidos

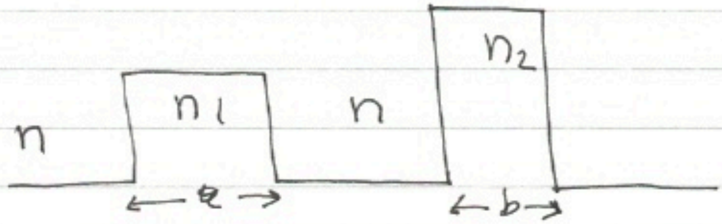



$\to$ # de modos depende la geometría y los índices de



La luz es transportada sin pérdidas a lo largo de la guía.

**<u>N = 2 GUÍAS</u>**




<u>Soluc</u>: ecuac. de Maxwell → plantear soluc. en los diferentes regiones y "pegarlas" adecuadamente.
⇒ ecuac. (trascendental) para los modos permitidos

<u>Más fácil</u>: Partir del límite donde las guías están <u>desacopladas</u>.

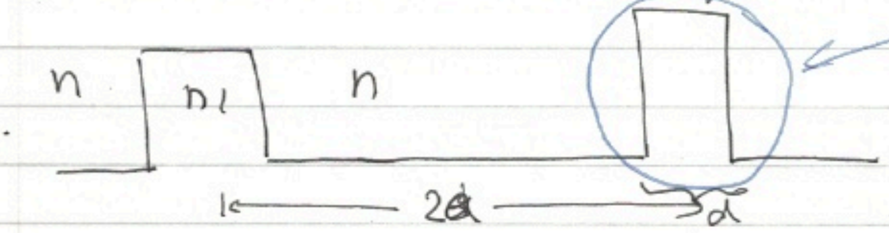
$$E_1(x,z) = a_1 u_1(x) e^{i\beta_1 z}$$

$$E_2(x,z) = a_2 u_2(x) e^{i\beta_2 z}$$

con $a_1, a_2$ <u>constantes</u>

Al acercar las guías, supondremos que el <u>único</u> cambio es que $a_1, a_2$ se hacen dep. de z, pero <u>débilmente</u>
$|da_1/dz| \ll 1/\beta_1$ y $|da_2/dz| \ll 1/\beta_2$

la guía 2 se puede ver como una PERTURBACIÓN del medio fuera de la guía 1.




región con índice $n_2-n$, loc. a distancia 2d y de ancho d

→ ~~densidad~~ cambio en la densidad de polarización

$$P = (\epsilon_2-\epsilon) E_2 = \epsilon_0 (n_2^2-n^2) E_2 = (k_2^2-k^2)E_2$$

$$\rightarrow \boxed{\nabla^2 E_1 + k_1^2 E_1 = -(k_2^2-k^2)E_2}$$

similar/. , la ec. de H. para la guía 2 , perturbada por la presencia de la guía 1 es

$$\boxed{\nabla^2 E_2 + k_2^2 E_2 = -(k_1^2-k^2)E_1}$$

Ponemos $E_1(x,z) = a_1(z)\, e_1(x,z)$

$E_2(x,z) = a_2(z)\, e_2(x,z)$

donde $e_1(x,z) = u_1(x) e^{i\beta_1 z}$ ; $e_2(x,z) = u_2(x) e^{i\beta_2 z}$

donde $e_1$ y $e_2$ deben satisfacer

$$\nabla^2 e_1 + k_1^2 e_1 = 0 \quad \wedge \quad \nabla^2 e_2 + k_2^2 e_2 = 0$$

donde $k_1 = \begin{cases} n_1 k_0 & \text{dentro guía 1} \\ n k_0 & \text{fuera guía 1} \end{cases}$

$$y \quad K_2 = \begin{cases} n_2 k_0 & \text{dentro guía } 2 \\ n k_0 & \text{fuera guía } 2 \end{cases}$$

sust. $E_1 = a_1 e_1$, y usando $\nabla^2(a_1 e_1) = a_1 \nabla^2 e_1 + e_1 \nabla^2 a_1 + 2\vec{\nabla} a_1 \cdot \vec{\nabla} e_1$

$$\Rightarrow \underline{a_1 \nabla^2 e_1} + \vec{\nabla} a_1 \cdot \vec{\nabla} e_1 + e_1 \nabla^2 a_1 + \underline{K_1^2 a_1 e_1} = -(K_2^2 - K^2) a_2 e_2$$

$$a_1 \underbrace{(\nabla^2 e_1 + K_1^2 e_1)}_{0} + 2 \frac{da_1}{dz}\frac{de_1}{dz} = -(K_2^2 - K^2) a_2 e_2$$

$$+ e_1 \nabla^2 a_1$$

$$\therefore \quad e_1 \frac{d^2 a_1}{dz^2} + 2 \frac{da_1}{dz}\frac{de_1}{dz} = -(K_2^2 - K^2) a_2 e_2$$

$\left|\frac{d^2 a_1}{dz^2}\right| \ll \beta_1 \left|\frac{da_1}{dz}\right|$ y usando $e_1 = u_1 e^{i\beta_1 z}$; $e_2 = u_2 e^{i\beta_2 z}$

$$\Rightarrow 2 \frac{da_1}{dz}(i\beta_1) u_1(x) e^{i\beta_1 z} = -(K_2^2 - K^2) a_2 u_2(x) e^{i\beta_2 z} \quad / u_1$$

$$2i\beta_1 \left(\frac{da_1}{dz}\right)\left(\int_{-\infty}^{\infty} u_1^2(x)\,dx\right) e^{i\beta_1 z} = -(K_2^2 - K^2) a_2 e^{i\beta_2 z} \int_{-\infty}^{\infty} u_1(x) u_2(x)\,dx$$

$$i \frac{da_1}{dz} = \frac{-(n_2^2 - n^2) k_0^2}{2\beta_1} \frac{\int u_1(x) u_2(x)\,dx}{\int u_1^2(x)\,dx} e^{i(\beta_2 - \beta_1) z} a_2$$

$$\boxed{i \frac{da_1}{dz} = V_{21} a_2 e^{-i\Delta\beta z}} \quad (1) \qquad \Delta\beta \equiv \beta_1 - \beta_2$$

similar'/, reempl. $E_2 = e_2 e_1$ conduce a:

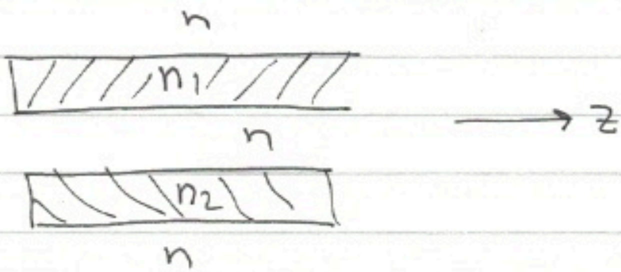
$$\boxed{i\frac{da_2}{dz} = V_{12}\, a_1\, e^{i\Delta\beta z}}, \qquad (2)$$

DINAMICA DISCRETA

donde $V_{12} \cong -\frac{(n_1^2-n^2)k_0^2}{2\beta_2}\,\frac{\int u_2(x)\,u_1(x)\,dx}{\int u_2^2(x)\,dx}$.

(1) y (2) definen una dinámica discreta de 2 sitios.

Ejemplo: $\Rightarrow$




tomo $d/dz$ en (1):

$$i\frac{d^2a_1}{dz^2} = V_{21}\left(\frac{da_2}{dz}\right)e^{-i\Delta\beta z} + V_{21}\,a_2(-i\Delta\beta)\,e^{-i\Delta\beta z}$$

$$= V_{21}(-i)V_{12}a_1\,\cancel{e^{i\Delta\beta z}}\cdot\cancel{e^{-i\Delta\beta z}} - i\Delta\beta V_{21}\,a_2\,e^{-i\Delta\beta z}$$

$$\to \frac{d^2a_1}{dz^2} + V_{12}V_{21}\,a_1 + \Delta\beta\frac{\cancel{V_{21}}}{\cancel{V_{21}}}(i)\frac{da_1}{dz} = 0$$

$$\Rightarrow a_1'' + i\Delta\beta\, a_1' + V_{12}V_{21}\,a_1 = 0 \qquad a_1 \sim e^{\lambda z}$$

$$\lambda^2 + i\Delta\beta\lambda + V_{12}V_{21} = 0 \Rightarrow \lambda = \underline{-i\Delta\beta \pm \sqrt{-\Delta\beta^2 - 4V_{12}V_{21}}}$$

$$\lambda = -i\frac{\Delta\beta}{2} \pm \frac{i}{2}\sqrt{\Delta\beta^2 + 4V_{12}V_{21}} \quad (3)$$

$$a_1(z) = e^{-i\frac{\Delta\beta}{2}z}\left(\alpha\cos\gamma z + \beta\sin\gamma z\right)$$

$$\gamma \equiv \sqrt{\left(\frac{\Delta\beta}{2}\right)^2 + V_{12}V_{21}} \quad (4)$$

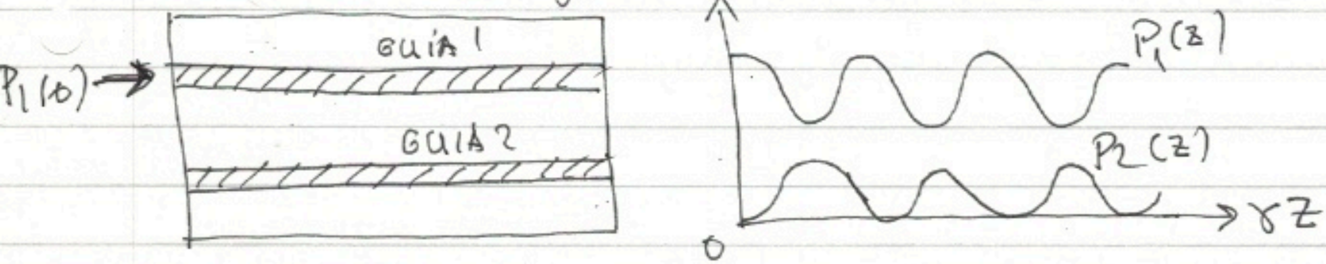
Si $a_1(0) \neq 0$ y $a_2(0) = 0$

$$\Rightarrow a_1(z) = a_1(0)\, e^{-i\frac{\Delta\beta}{2}z}\left[\cos(\gamma z) + i\frac{\Delta\beta}{2\gamma}\sin(\gamma z)\right]$$

$$a_2(z) = i a_1(0)\frac{V_{12}}{\gamma}\, e^{i\frac{\Delta\beta}{2}z}\sin(\gamma z)$$

<u>Potencia óptica</u> $\quad P_1(z) \propto |a_1(z)|^2 \;;\; P_2(z) \propto |a_2(z)|^2$

$$\left.\begin{aligned}\Rightarrow P_1(z) &= P_1(0)\left[\cos^2(\gamma z) + \left(\frac{\Delta\beta}{2\gamma}\right)^2\sin^2(\gamma z)\right]\\ P_2(z) &= P_1(0)\frac{V_{12}^2}{\gamma^2}\sin^2(\gamma z)\end{aligned}\right\}\ (5)$$

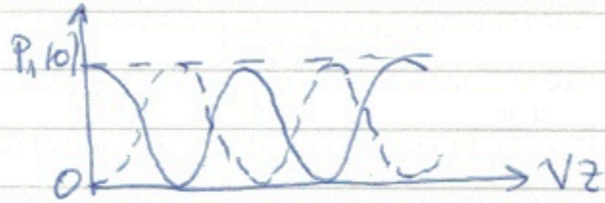



la potencia es intercambiada <u>periódicamente</u>, con periodo $2\pi/\gamma$

EJ] Dem que $|a_1(z)|^2 + |a_2(z)|^2$ es cantidad <u>conservada</u> por la dinámica.

Caso de guías idénticas: $n_1 = n_2 \Rightarrow \beta_1 = \beta_2 \Rightarrow \Delta\beta = 0$
$\Rightarrow V_{12} = V_{21} \equiv V$

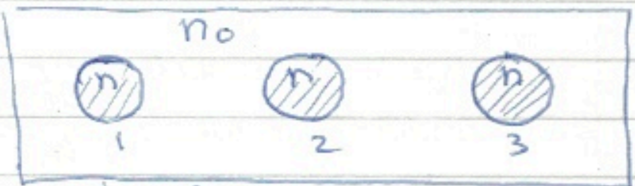
$$\Rightarrow P_1(z) = P_1(0)\cos^2(Vz)$$
$$P_2(z) = P_1(0)\sin^2(Vz)$$




completo intercambio de energía entre las guías de onda.

Para $z = L_0 = \frac{\pi}{2V}$ $P_1 = 0$ y $P_2 = P_1(0)$

long. de acoplamiento.




## 3 GUÍAS IDÉNTICAS

sup. c/guía monomodal y equidistante, y sólo consideramos interacciones a primeros vecinos.

$V_{12} = V_{21} = V_{23} = V_{32} \equiv V$
$V_{13} = V_{31} \simeq 0$ (acoplamiento débil)

$$i\frac{da_1}{dz} = Va_2$$
$$i\frac{da_2}{dz} = V(a_1 + a_3)$$
$$i\frac{da_3}{dz} = Va_2$$

condic. inicial: $a_1(0) = 1$
$a_2(0) = 0 = a_3(0)$.

una vez calculados $a_1(z), a_2(z), a_3(z)$

$$E(x,z) = a_1(z)u_1(x-x_1)e^{i\beta z} + a_2(z)u_2(x-x_2)e^{i\beta z} + a_3(z)u_3(x-x_3)e^{i\beta z}$$

$\Rightarrow$