



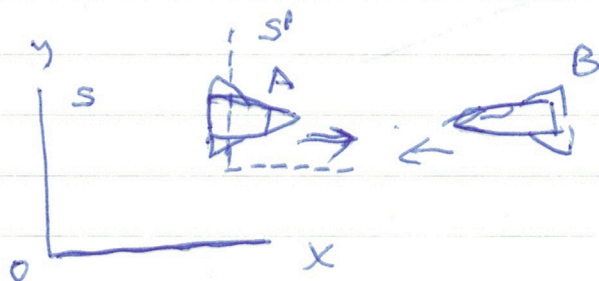
En gal,
$$\vec{v}_A = \frac{\vec{v}_B + \vec{v}_{A'}}{1 + \frac{\vec{v}_A \cdot \vec{v}_B}{c^2}} \Rightarrow$$

$$\vec{v}_{A'} = \frac{\vec{v}_A - \vec{v}_B}{1 - \frac{\vec{v}_A \cdot \vec{v}_B}{c^2}} \quad \text{In our case } \begin{aligned} \vec{v}_A &= -0.5c \hat{x} \\ \vec{v}_B &= -0.8c \hat{x} \end{aligned}$$

$$\Rightarrow v_{A'} = \frac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$$

Ej. 2 naves A y B se mueven en direcciones opuestas

⑤ (tierra) $V_A = 0.750c$
 $V_B = 0.850c$



Hallar la veloc. de B c/n a A.

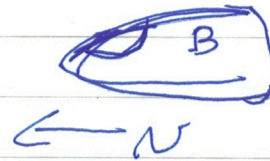
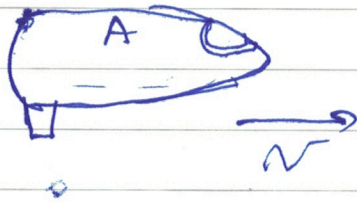
Solución tomar S' en la nave A, con $U = 0.750c$

La veloc. entre S y S' . La nave B se toma como un objeto moviéndose hacia la izquierda con velocidad $u_x = -0.850c$ c/n a la tierra (S)

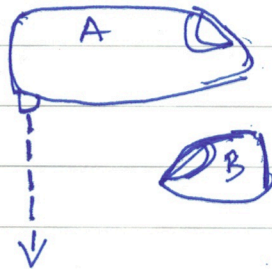
$\Rightarrow$ la veloc. de B c/n a A (S') será

$$u_{x'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

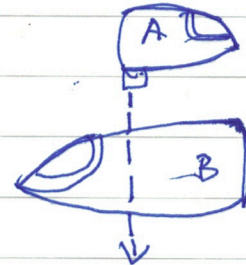
Típica paradoja



(i)

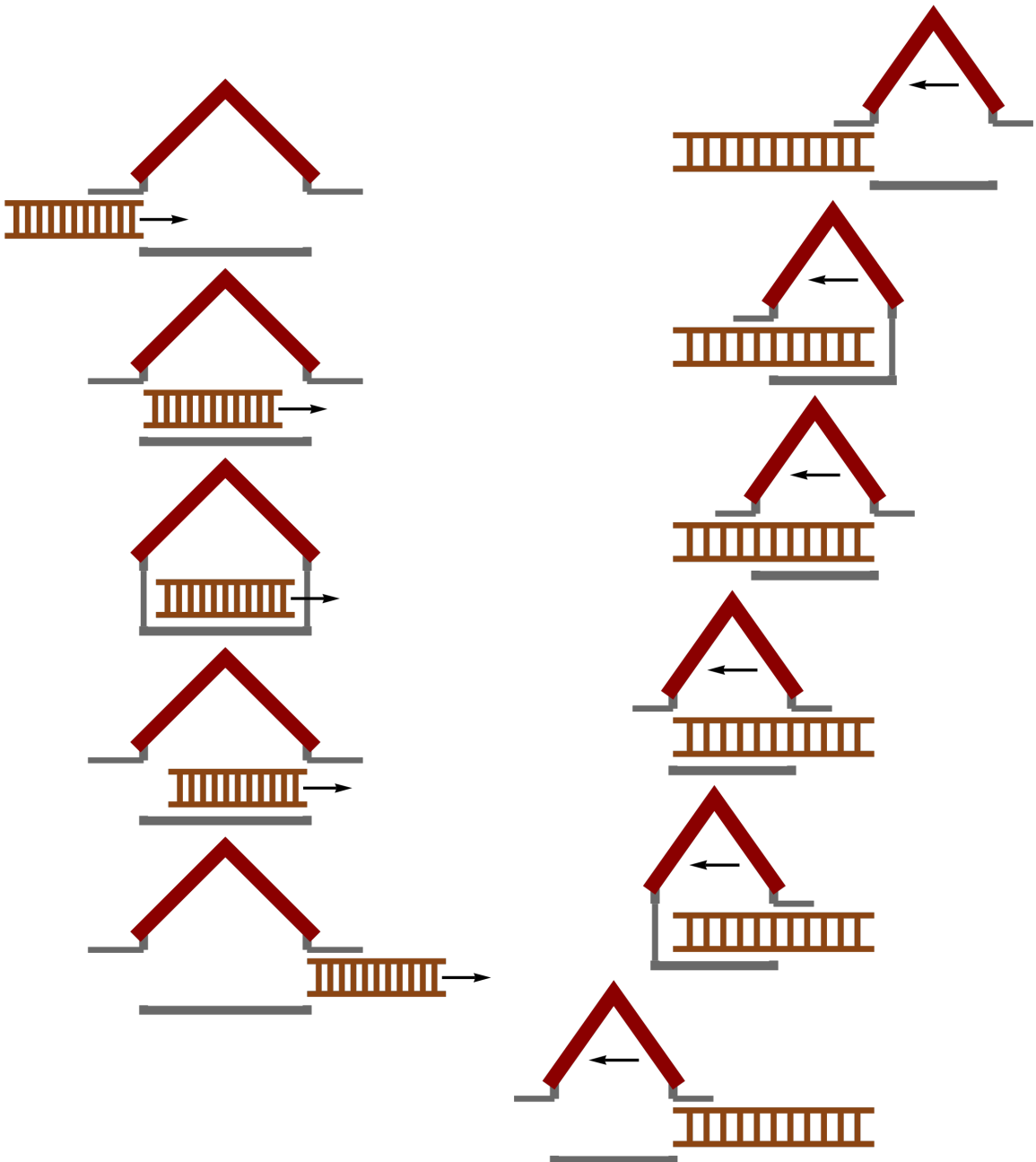
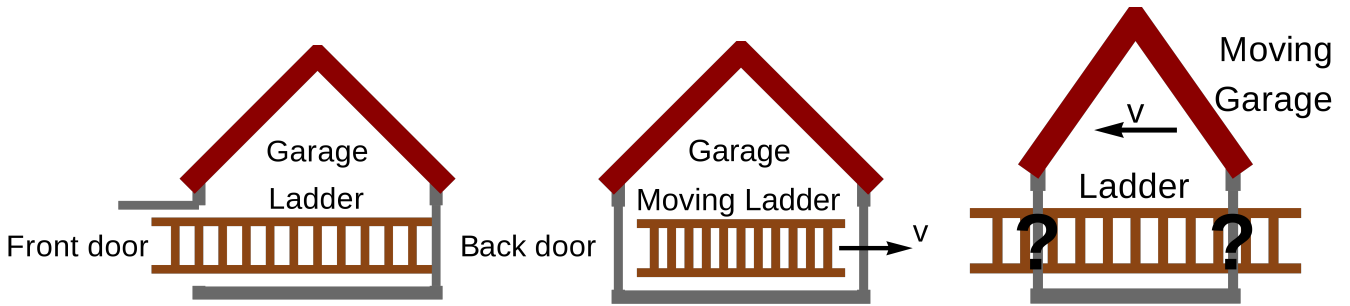


(ii')



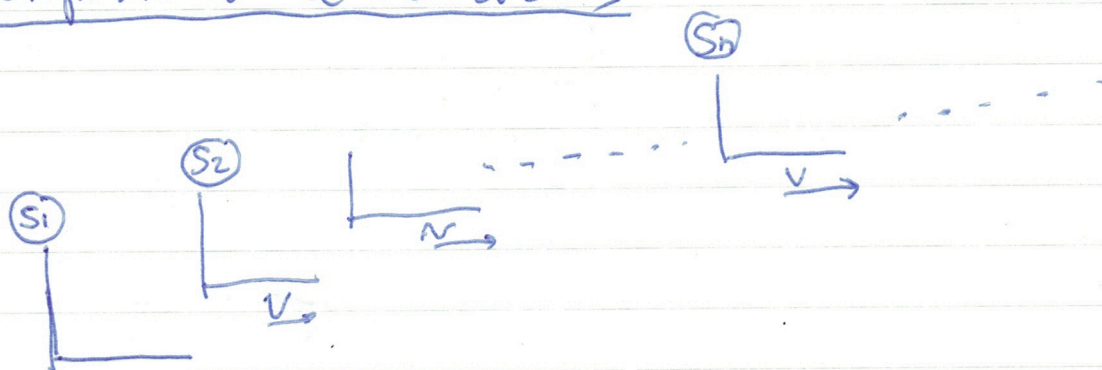
ES DESTRUIDA LA NAVE B ?

Paradoja de la escalera



Simultaneidad !

composição de velocidade



$$u_n =$$

$$\frac{u_{n+1} + v}{1 + \frac{u_{n+1}v}{c^2}}$$

para $n \rightarrow \infty$, $u_n \rightarrow u$, $u_{n+1} \rightarrow u$

$$u = \frac{u + v}{1 + \frac{uv}{c^2}} \Rightarrow u \left(1 + \frac{uv}{c^2}\right) = u + v$$

$$u^2 \frac{v}{c^2} = v \Rightarrow u^2 = c^2$$

$$\boxed{u \rightarrow c}$$

Muons

$$N(t) = N_0 e^{-t/\tau}$$

τ = vida media $\approx 2\mu s$ (est. reposo en el muón)
 $v \approx 0.998c$ y son creados en la alta atmósfera.

la distancia recorrida (desde S) debería ser $h \approx v\tau = 0.998c \cdot 2\mu s = 600 \text{ m}$.

Desde la Tierra, $\tau \rightarrow \gamma\tau \approx 15 \times 2\mu s = 30\mu s$
 $\Rightarrow h \approx 9.000 \text{ m}$

Desde el muón, τ no cambia, pero el suelo se aproxima a $0.998c \Rightarrow$ se contrae la altura
y $9000 \text{ m} \rightarrow \frac{9000 \text{ m}}{\gamma} = \frac{9000}{15} = 600 \text{ m}$.

$\therefore$ El muón llega al suelo ya sea realizando desde S a S'.

Prob 26 (Setway) $\rightarrow$

Two objects A and B are moving along the x-axis. Object A has length L and object B has length $3L$. They are moving with velocities v_A and v_B respectively. The Lorentz factor is given by $\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$.

We are given that the objects are moving such that their proper lengths are equal in the lab frame. That is, $L_A = L_B$.

$$L_A = L_B \Rightarrow L = 3L \gamma_B \gamma_A$$

Since the proper lengths are equal, we have $L_A = L_B$. This implies that the Lorentz factors must satisfy $\gamma_A = 3\gamma_B$.

$$\gamma_A = 3\gamma_B$$

Using the definition of the Lorentz factor, we can write $\gamma_A = \frac{1}{\sqrt{1 - (v_A/c)^2}}$ and $\gamma_B = \frac{1}{\sqrt{1 - (v_B/c)^2}}$.

Substituting $\gamma_A = 3\gamma_B$ into the equation for γ_A , we get $\frac{1}{\sqrt{1 - (v_A/c)^2}} = 3 \frac{1}{\sqrt{1 - (v_B/c)^2}}$.

$$\Rightarrow \sqrt{1 - (v_B/c)^2} = 3 \sqrt{1 - (v_A/c)^2}$$

$$1 - (v_B/c)^2 = 9 \left(1 - (v_A/c)^2 \right)$$

$$\Rightarrow 1 - (v_B/c)^2 = 9 - 9(v_A/c)^2 \Rightarrow (v_B/c)^2 = 8 - 9(v_A/c)^2$$

$$\Rightarrow \frac{v_B}{c} = \sqrt{8 - 9\left(\frac{v_A}{c}\right)^2}$$

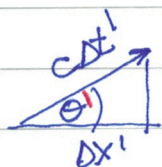
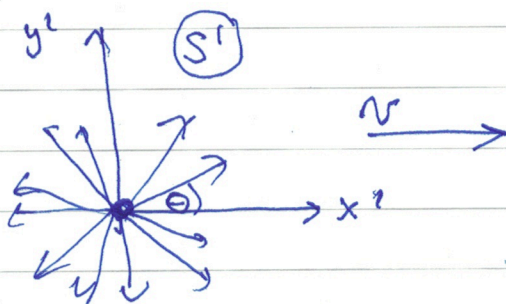
$$1 - \left(\frac{v_B}{c}\right)^2 = \frac{1}{9} \left(1 - \left(\frac{v_A}{c}\right)^2 \right) = \frac{1}{9} - \frac{1}{9} \left(\frac{v_A}{c}\right)^2$$

$$\frac{8}{9} - \left(\frac{v_B}{c}\right)^2 = \frac{1}{9} \left(\frac{v_A}{c}\right)^2 \Rightarrow \left(\frac{v_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9} \left(\frac{v_A}{c}\right)^2$$

$$\frac{v_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9} \left(\frac{v_A}{c}\right)^2} < 1$$

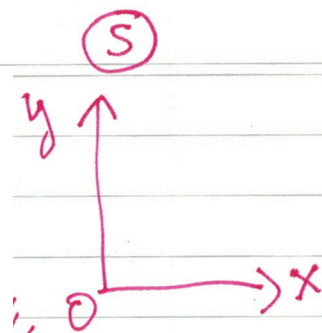
$$= 0.95$$

Meças de luz



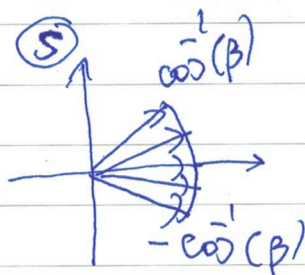
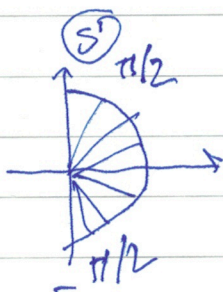
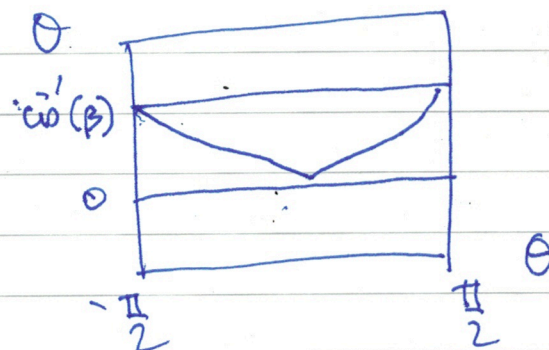
$$\cos \theta' = \frac{dx'}{c dt'}$$

$$\cos \theta = \frac{dx}{c dt}$$



$$\begin{aligned} \cos \theta &= \frac{dx}{c dt} = \frac{\gamma (dx' + v dt')}{\gamma (dt' + \frac{v}{c^2} dx')} = \frac{\left(\frac{dx'}{dt'}\right) + v}{c \left(1 + \frac{v}{c^2} \frac{dx'}{dt'}\right)} \\ &= \frac{\frac{dx'}{c dt'} + \frac{v}{c}}{1 + \frac{v}{c} \frac{dx'}{c dt'}} = \frac{\cos \theta' + \frac{v}{c}}{1 + \left(\frac{v}{c}\right) \cos \theta'} \end{aligned}$$

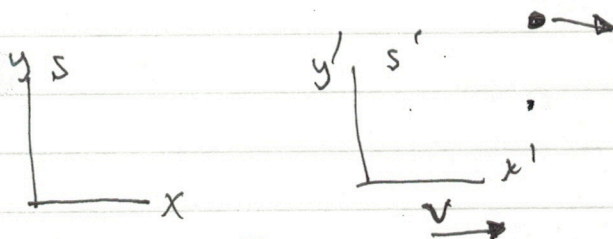
$$\boxed{\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'}}$$



Movimiento Acelerado

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}; \quad u_y = \frac{u_y' / \gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma \left(t' + \frac{v x'}{c^2} \right)$$



$$\begin{aligned} du_x &= \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2} du_x' \right] \\ &= \frac{1 + \frac{v u_x'}{c^2} - \frac{u_x' v}{c^2} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2} du_x' = \frac{(1 - \left(\frac{v}{c}\right)^2) du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2} \end{aligned}$$

también $dt = \gamma \left(dt' + \frac{v}{c^2} dx' \right) = \gamma dt' \left(1 + \frac{v u_x'}{c^2} \right)$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{(du_x'/dt')}{\gamma^3 \left(1 + \frac{v u_x'}{c^2} \right)^3}$$

$$\therefore \boxed{a_x = \frac{a_{x'}}{\gamma^3 \left[1 + \frac{v u_{x'}}{c^2} \right]^3}}$$

similantemente [Ejercicio]

$$a_y = \frac{a_{y'}}{\gamma^2 \left(1 + \frac{v u_{x'}}{c^2} \right)^2} - \frac{(v u_{y'}/c^2) a_{x'}}{\gamma^2 \left(1 + \frac{v u_{x'}}{c^2} \right)^3}$$

Esempio cinematica relativistica

(S)

(S')



$$a_{x'} = g$$

$$dx' = 0$$

$$a_x = \frac{a_{x'}}{\gamma^3 (1 + v \frac{u_{x'}}{c^2})^3} = \frac{g}{\gamma^3 (u_x)}$$

$$\frac{du_x}{dt} = g \left(1 - \left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\int_0^{u_x} \frac{du_x}{\left(1 - \left(\frac{u_x}{c}\right)^2\right)^{3/2}} = \int_0^t g dt$$

$$\frac{u_x}{\sqrt{1 - \left(\frac{u_x}{c}\right)^2}} = gt$$

$$\Rightarrow \frac{u^2}{1 - \frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2 \frac{u^2}{c^2}$$

$$u^2 \left(1 + \frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1 + \frac{(gt)^2}{c^2}} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1 + (gt/c)^2}}} \quad (*)$$

$$\text{for } u = c/2 \Rightarrow \left(\frac{c}{2}\right)^2 \left(1 + \frac{(gt)^2}{c^2}\right) = (gt)^2 \Rightarrow \left(\frac{c}{2}\right)^2 = (gt)^2 - \frac{(gt)^2}{4} = \frac{3}{4} (gt)^2$$

$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3} gt$$

$$(*) \Rightarrow x(t) = \frac{c^2}{g} \left[-1 + \sqrt{1 + (gt/c)^2}\right]$$

$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}g}} = 6.8 \text{ months}$$

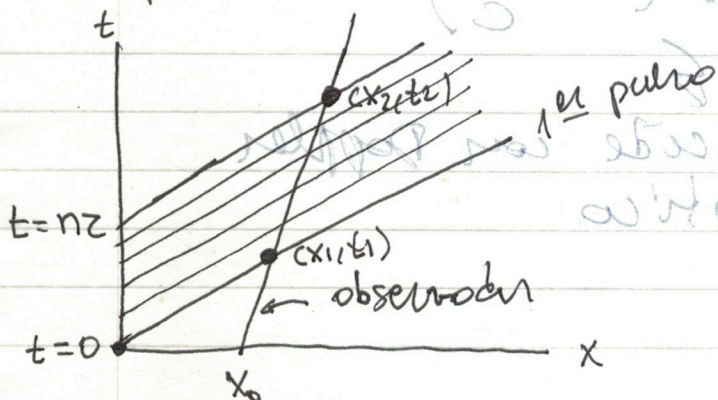
$$\text{si } c \gg 1, x \rightarrow \frac{1}{2} gt^2$$

Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau \Rightarrow$ en S, la frec. medida es $\nu = 1/\tau$

El 1er pulso es emitido en $t=0$, cuando el receptor (S') está en $x = x_0$



$$x_1 = ct_1 = x_0 + vt_1$$

$$x_2 = c(t_2 - n\tau) = x_0 + vt_2$$

$$t_2 - t_1 = \frac{cn\tau}{c - v}$$

$$x_2 - x_1 = \frac{v cn\tau}{c - v}$$

En S': $t_2' - t_1' = \gamma \left[(t_2 - t_1) - \frac{v}{c^2} (x_2 - x_1) \right]$

$$= \gamma \left(\frac{cn\tau}{c - v} - \frac{v}{c^2} \cdot \frac{v cn\tau}{c - v} \right) = \frac{\gamma cn\tau}{c - v} \left(1 - \frac{v^2}{c^2} \right)$$

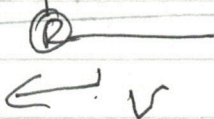
$$\Rightarrow \lambda' = \frac{\gamma cn\tau}{c - v} \left(1 - \frac{v^2}{c^2} \right) = \frac{\gamma \lambda}{1 - \frac{v}{c}} \left(1 - \frac{v^2}{c^2} \right) = \frac{\lambda}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{1}{1 - \frac{v}{c}}$$

$$= \frac{\lambda \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}} = \lambda \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

$$\Rightarrow \boxed{\nu' = \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2} \nu}$$

Doppler longitudinal $\rightarrow$

Esercizio Doppler relativistico



$$\lambda' = \lambda \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2}$$

con $v/c \ll 1$

$$\lambda' \approx \lambda \left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} + \left(\frac{v}{c} \right)^2 \right)^{1/2}$$

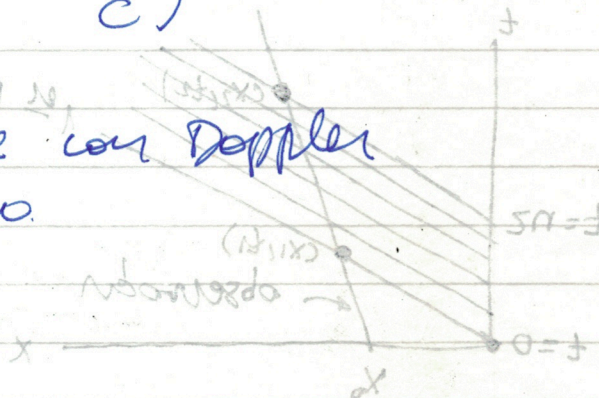
$$\approx \lambda \left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} \right) = \lambda \left(1 - \frac{v}{c} \right)^2$$

$$\lambda' = \lambda \left(1 - \frac{v}{c} \right)$$

coincide con Doppler
acustico

$$\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$$

$$\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$$



$$\lambda' = \lambda \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2}$$

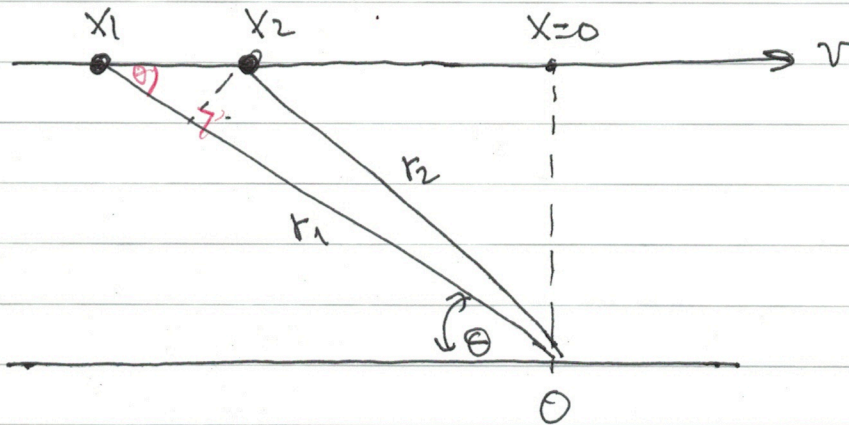
$$\lambda' = \lambda \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2}$$

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Doppler relativistico

Doppler Transversal



2 pulsos sucesivos son emitidos en $x=x_1$ y $x=x_2$ en los instantes $t=t_1$ y $t=t_2$.

En el sist. en reposo c/n al satélite, el intervalo entre pulsos es τ . $\Rightarrow t_2 - t_1 = \gamma \tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O

" " #2 " r_2/c " " " O

$$\Rightarrow \text{Intervalo entre pulsos: } \tau' = t_2 + \frac{r_2}{c} - (t_1 + \frac{r_1}{c})$$

$$\text{Si } |x_2 - x_1| \ll r_1 \Rightarrow r_1 - r_2 \approx (x_2 - x_1) \cos \theta$$

$$= (vt_2 - vt_1) \cos \theta = v(t_2 - t_1) \cos \theta = v \gamma \tau \cos \theta$$

$$\therefore \tau' = (t_2 - t_1) + \frac{1}{c}(r_1 - r_2) = \gamma \tau - \frac{v \gamma \tau \cos \theta}{c} = \gamma \tau (1 - \frac{v}{c} \cos \theta)$$

$$\Rightarrow \nu' = \frac{\nu}{\gamma (1 - \frac{v}{c} \cos \theta)} = \boxed{\frac{\nu (1 - (v/c)^2)^{1/2}}{(1 - (v/c) \cos \theta)}}$$