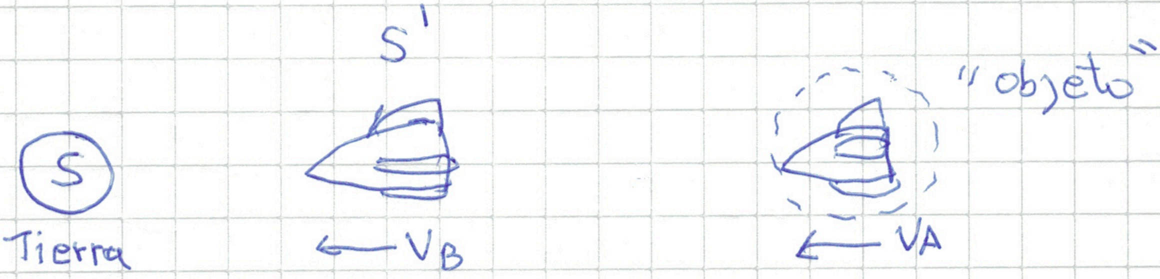


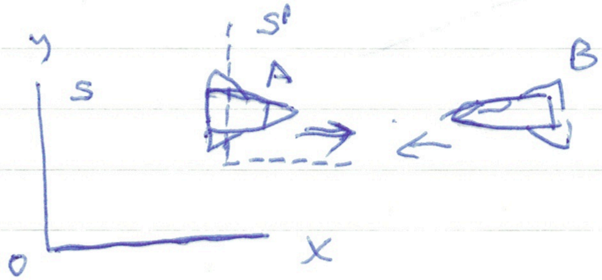
En gral, $\vec{v}_A = \dfrac{\vec{v}_B + \vec{v}_A'}{1 + \dfrac{\vec{v}_A' \cdot \vec{v}_B}{c^2}} \Rightarrow$

$\vec{v}_A' = \dfrac{\vec{v}_A - \vec{v}_B}{1 - \dfrac{\vec{v}_A \cdot \vec{v}_B}{c^2}}$. in our case $\vec{v}_A = -0.5c\,\hat{i}$
$\vec{v}_B = -0.8c\,\hat{i}$

$$\Rightarrow v_A' = \frac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$$

EJ: 2 naves A y B se mueven en direcciones opuestas

(S) (tierra) $V_A = 0.750c$
$V_B = 0.850c$



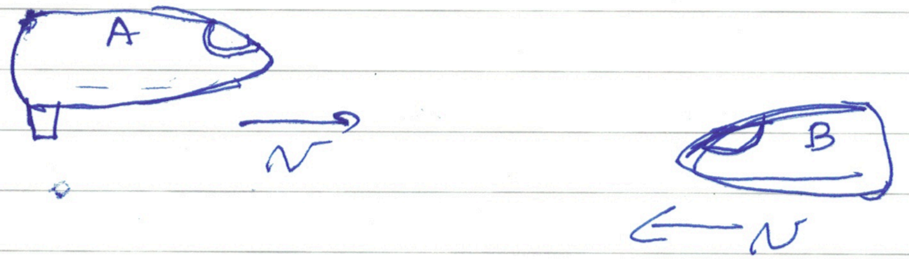
Hallar la veloc. de B c/r a A.

Solución: tomar S' en la nave A, con $v = 0.750c$ la veloc. entre S y S'. La nave B se toma como un objeto moviéndose hacia la izquierda con velocidad $u_x = -0.850c$ c/r a la tierra (S)

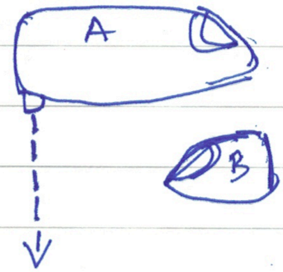
⇒ la veloc. de B c/r a A (S') será

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.9771c$$

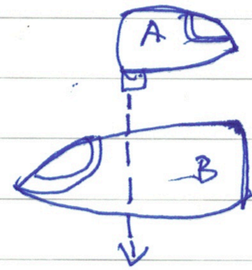
Típica paradoja



(i)

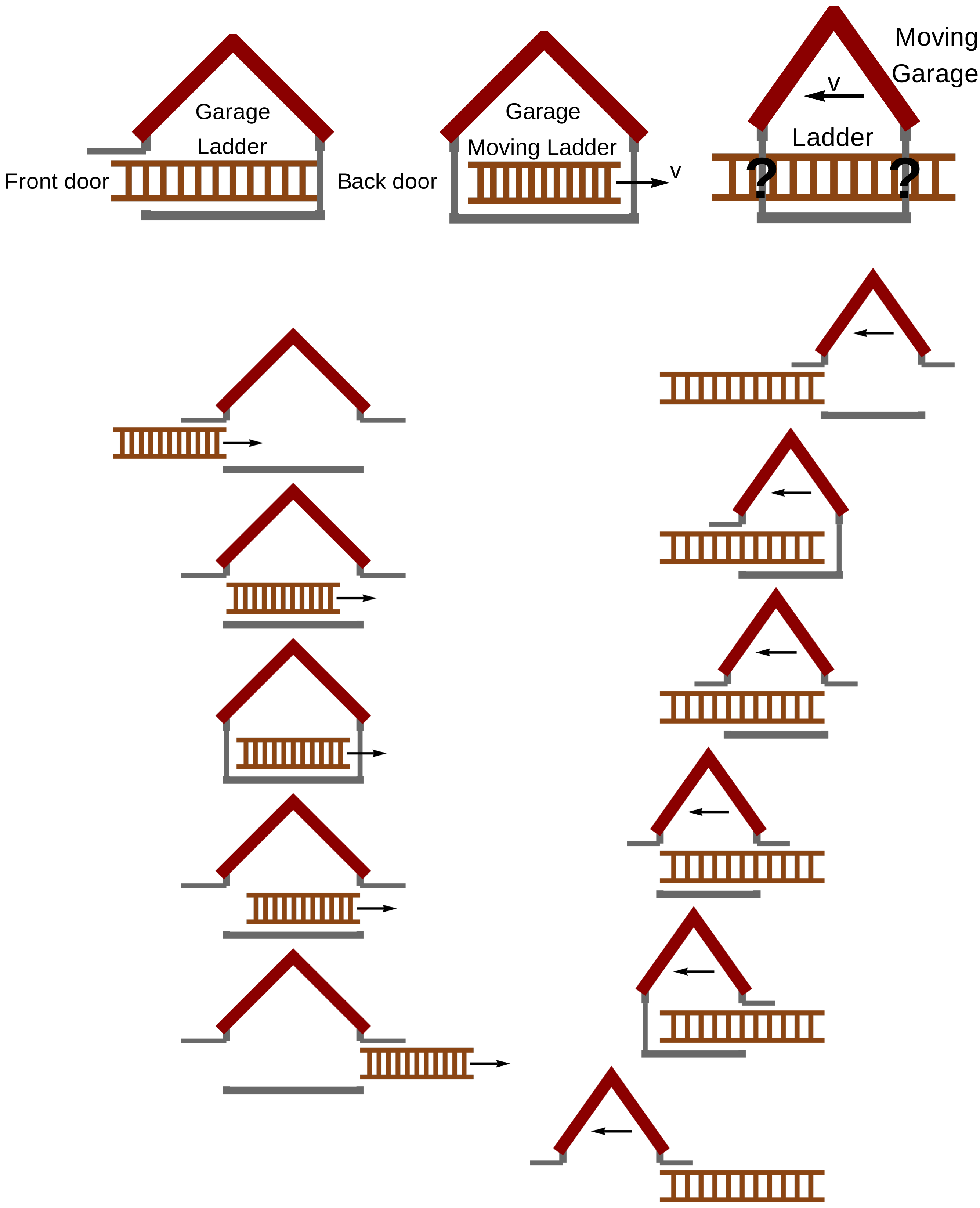


(ii)



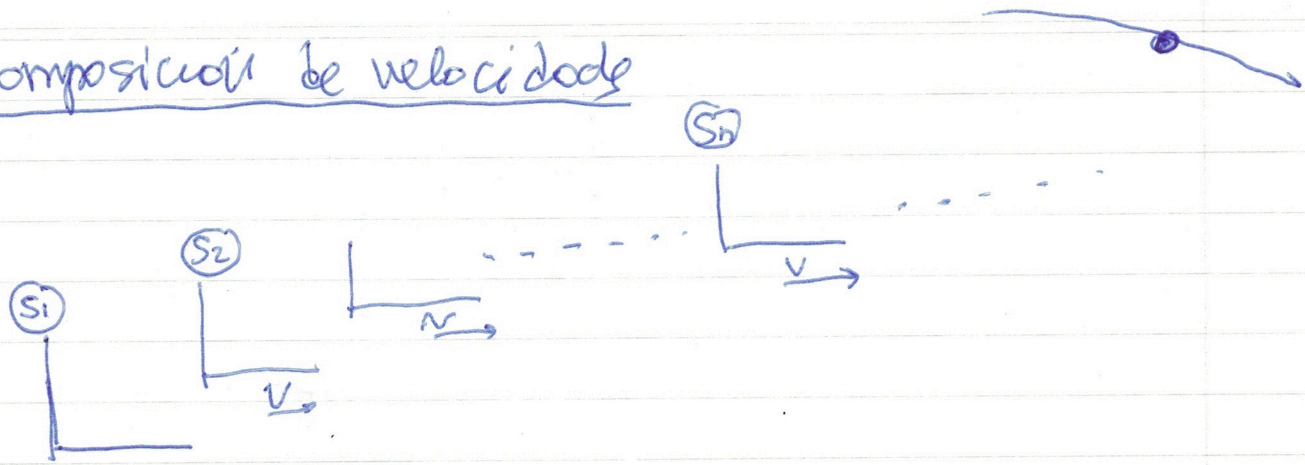
ES DESTRUIDA LA NAVE B ?

Paradoja de la escalera



Simultaneidad !

<u>composición de velocidades</u>



$$u_n = \frac{u_{n+1} + v}{1 + \frac{u_{n+1} V}{c^2}}$$

para $n \to \infty$, $u_n \to u$, $u_{n+1} \to u$

$$u = \frac{u + V}{1 + \frac{uV}{c^2}} \Rightarrow u\left(1 + \frac{uV}{c^2}\right) = u + V$$

$$\frac{u^2 V}{c^2} = V \Rightarrow u^2 = c^2$$

$$\boxed{u \to c}$$

Muones

$$N(t) = N_0 e^{-t/\tau}$$

τ = vida media $\simeq 2\mu s$ (sist. reposo c/r al muón)
$v \simeq 0.998c$ y son creados en la alta atmósfera.

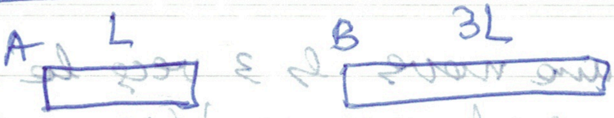
La distancia recorrida (desde S) debería ser $h \simeq v\tau = 0.998c \cdot 2\mu s = 600$ m.

Desde la tierra, $\tau \to \gamma\tau \simeq 15 \times 2\mu s = 30\mu s$
$\Rightarrow h \simeq 9.000$ m

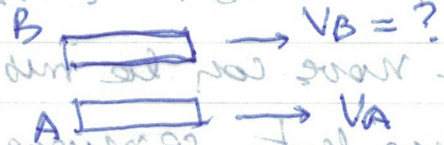
Desde el muón, τ no cambió, pero el suelo se aproxima a $0.998c$ $\Rightarrow$ se contrae la altura y $9000 m \to \frac{9000}{\gamma} m = \frac{9000}{15} = 600$ m.

$\therefore$ El muón llega al suelo ya sea analizado desde S ó S'.

Prob. 26 (Serway) $\rightarrow$



$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$



S ..
$$L_A = \frac{L}{\gamma_A}$$

$$L_B = \frac{3L}{\gamma_B}$$

but $L_A = L_B \Rightarrow 1 = \dfrac{\frac{L}{\gamma_A}}{\frac{3L}{\gamma_B}} = \dfrac{\cancel{L}\gamma_B}{\gamma_A \cdot 3\cancel{L}} = \dfrac{\gamma_B}{3\gamma_A}$

$$\Rightarrow \gamma_B = 3\gamma_A \Rightarrow \frac{1}{\gamma_B} = \frac{1}{3\gamma_A} \Rightarrow \sqrt{1-\left(\frac{V_B}{c}\right)^2} = \frac{1}{3}\sqrt{1-\left(\frac{V_A}{c}\right)^2}$$

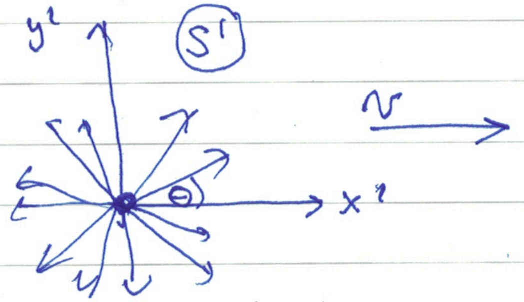
$$1-\left(\frac{V_B}{c}\right)^2 = \frac{1}{9}\left(1-\left(\frac{V_A}{c}\right)^2\right) = \frac{1}{9} - \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

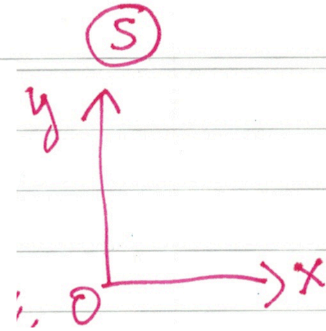
$$\frac{8}{9} - \left(\frac{V_B}{c}\right)^2 = -\frac{1}{9}\left(\frac{V_A}{c}\right)^2 \Rightarrow \left(\frac{V_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

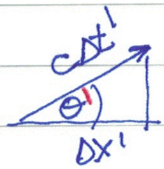
$$\frac{V_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2} < 1$$

$$= 0.95$$

Haces de luz





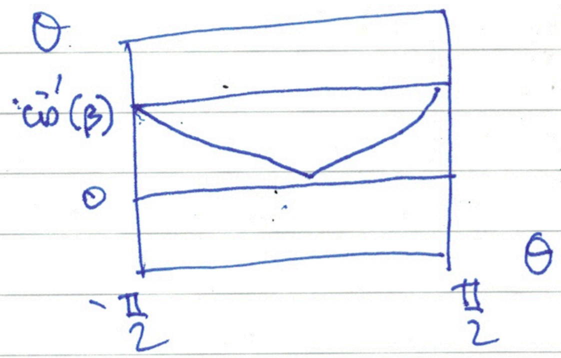


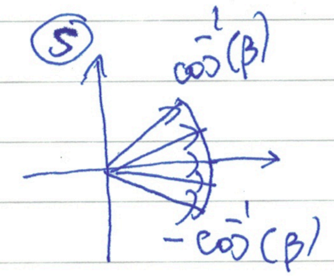
$$\cos\theta' = \frac{\Delta x'}{c\Delta t'} \qquad\qquad \cos\theta = \frac{\Delta x}{c\Delta t}$$

$$\cos\theta = \frac{\Delta x}{c\Delta t} = \frac{\cancel{\gamma}(\Delta x' + v\Delta t')}{c\gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)} = \frac{\left(\frac{\Delta x'}{\Delta t'}\right) + v}{c\left(1 + \frac{v}{c^2}\frac{\Delta x'}{\Delta t'}\right)}$$

$$= \frac{\frac{\Delta x'}{c\Delta t'} + \frac{v}{c}}{1 + \frac{v}{c}\frac{\Delta x'}{c\Delta t'}} = \frac{\cos\theta' + v/c}{1 + \left(\frac{v}{c}\right)\cos\theta'}$$

$$\boxed{\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}}$$

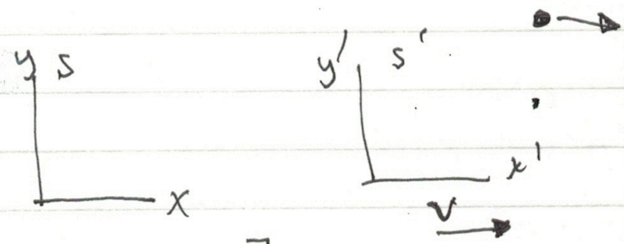




Movimientos Acelerados

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} \; ; \qquad u_y = \frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma \left(t' + \frac{v x'}{c^2}\right)$$



$$du_x = \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2} du_x'\right]$$

$$= \frac{\cancel{1} + \cancel{\frac{v u_x'}{c^2}} - \cancel{u_x' \frac{v}{c^2}} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2} du_x' = \frac{\left[1 - \left(\frac{v}{c}\right)^2\right] du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2}$$

también $\quad dt = \gamma \left(dt' + \frac{v}{c^2} dx'\right) = \gamma \, dt' \left(1 + \frac{v u_x'}{c^2}\right)$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{(du_x'/dt')}{\gamma^3 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

$$\therefore \boxed{a_x = \frac{a_x'}{\gamma^3 \left[1 + \frac{v u_x'}{c^2}\right]^3}}$$

similarmente [Ejercicio]

$$a_y = \frac{a_y'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^2} - \frac{(v u_y'/c^2)\, a_x'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^3}.$$

Ejemplo cinemática relativista

(S')

$a_{x'} = g$

$u_{x'} = 0$

(S)

$$a_x = \frac{a_{x'}}{\gamma^3\left(1+\frac{v u_{x'}}{c^2}\right)^3} = \frac{g}{\gamma^3(u_x)}$$

$$\frac{du_x}{dt} = g\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\underbrace{\int_0^{u_x} \frac{du_x}{\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}}}_{\frac{u_x}{\sqrt{1-\left(\frac{u_x}{c}\right)^2}}} = \int_0^t g\,dt$$

$$\frac{u_x}{\sqrt{1-\left(\frac{u_x}{c}\right)^2}} = gt$$

$$\Rightarrow \frac{u^2}{1-\frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2\frac{u^2}{c^2}$$

$$u^2\left(1+\frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1+\left(\frac{gt}{c}\right)^2} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1+(gt/c)^2}}} \quad (*)$$

Si $u = c/2$

$$\Rightarrow \left(\frac{c}{2}\right)^2\left(1+\left(\frac{gt}{c}\right)^2\right) = (gt)^2 \Rightarrow (c/2)^2 = (gt)^2 - \left(\frac{gt}{2}\right)^2 = \frac{3}{4}(gt)^2$$

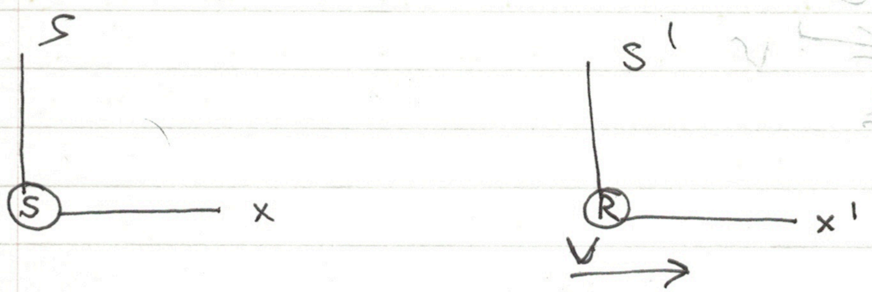
$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3}\,gt$$

$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}g}} = 6.8 \text{ months}$$

$$(*) \Rightarrow X(t) = \frac{c^2}{g}\left[-1+\sqrt{1+(gt/c)^2}\right]$$

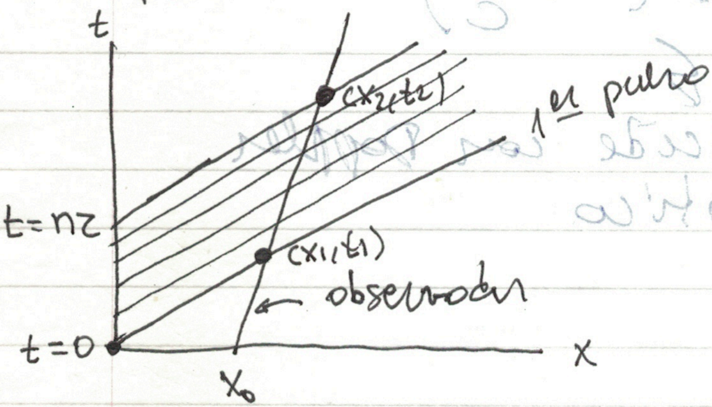
si $c \gg 1$, $X \to \frac{1}{2}gt^2$

Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau$ $\Rightarrow$ en S, la frec. medida es $\nu = 1/\tau$

El 1er pulso es emitido en $t=0$, cuando el receptor (S') esté en $x = x_0$



$$x_1 = c t_1 = x_0 + v t_1$$
$$x_2 = c(t_2 - n\tau) = x_0 + v t_2$$
$$\Rightarrow t_2 - t_1 = \frac{cn\tau}{c-v}$$
$$x_2 - x_1 = \frac{vcn\tau}{c-v}$$

En S': $$t_2' - t_1' = \gamma\left[(t_2-t_1) - \frac{v}{c^2}(x_2-x_1)\right]$$
$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v}{c^2}\cdot\frac{vcn\tau}{c-v}\right) = \frac{\gamma c\tau n}{c-v}\left(1-\frac{v^2}{c^2}\right)$$

$$\Rightarrow \tau' = \frac{\gamma c\tau}{c-v}\left(1-\frac{v^2}{c^2}\right) = \frac{\gamma\tau}{1-\left(\frac{v}{c}\right)}\left(1-\frac{v^2}{c^2}\right) = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\left(1-\frac{v^2}{c^2}\right)\cdot\frac{1}{1-\frac{v}{c}}$$
$$= \frac{\tau\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \tau\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \Rightarrow \boxed{\nu' = \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}\nu}$$

Doppler longitudinal →

$$\nu' = \nu \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2}$$

caso $v/c \ll 1$

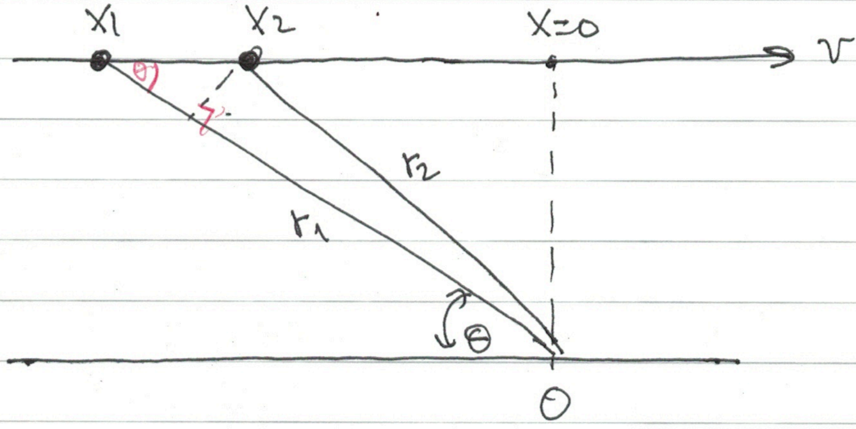
$$\nu' \approx \nu \left[\left(1 - \frac{v}{c}\right)\left(1 - \frac{v}{c} + \left(\frac{v}{c}\right)^2 - \dots\right) \right]^{1/2}$$

$$\approx \nu \left[\left(1 - \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) \right]^{1/2} = \nu \left(1 - \frac{v}{c}\right)$$

$\Downarrow$

· coincide con Doppler acústico

Doppler Transversal



2 pulsos sucesivos son emitidos en $x = x_1$ y $x = x_2$
en los instantes $t = t_1$ y $t = t_2$.
En el sist. en reposo c/r al satélite, el intervalo
entre pulsos es τ. $\Rightarrow t_2 - t_1 = \gamma\tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O
" " #2 " r_2/c " " " O

$\Rightarrow$ Intervalo entre pulsos: $\tau' = t_2 + \frac{r_2}{c} - (t_1 + r_1/c)$

si $|x_2 - x_1| \ll r_1 \Rightarrow r_1 - r_2 \simeq (x_2 - x_1)\cos\theta$

$= (v t_2 - v t_1)\cos\theta = v(t_2 - t_1)\cos\theta = v\gamma\tau\cos\theta$

$\therefore \tau' = (t_2 - t_1) + \frac{1}{c}(r_1 - r_2) = \gamma\tau - \gamma\frac{v}{c}\tau\cos\theta = \gamma\tau(1 - \frac{v}{c}\cos\theta)$

$$\Rightarrow \nu' = \frac{\nu}{\gamma(1 - \frac{v}{c}\cos\theta)} = \boxed{\frac{\nu\,(1 - (v/c)^2)^{1/2}}{(1 - (v/c)\cos\theta)}}$$