Ec. de transformación (entre S y S')

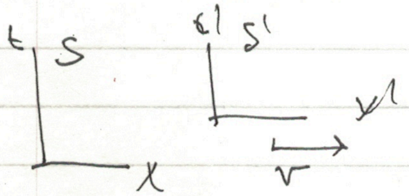
$x' = L_1(x,t)$

$t' = L_2(x,t)$

L_1, L_2 funcs. LINEALES (Debido a que un obj. a veloc. cte. en S, debiera también tener veloc. cte. en S')

$$x = ax' + bt' \quad \text{con} \quad x' = ax - bt$$

debe reducirse al caso Galileano a bajas velocidades



$x = 0 \Rightarrow ax' + bt' = 0 \Rightarrow ax' + bt' = 0 \Rightarrow \frac{x'}{t'} = -\frac{b}{a} = \boxed{-v}$ ← vel de S

$x' = 0 \Rightarrow ax - bt = 0 \Rightarrow ax = bt \to \frac{x}{t} = \frac{b}{a} = \boxed{v}$

veloc. de S'

$$\boxed{v = b/a}$$

Si una señal de luz se origina en el origen ~~común~~ de S: $x = ct$

Desde S': $x' = ct'$

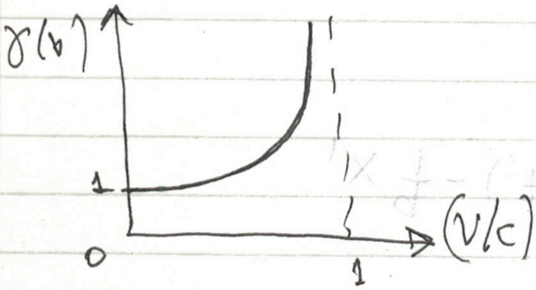
$$\Rightarrow \begin{array}{l} ct = act' + bt' \\ ct' = act - bt \end{array} \Rightarrow t' = (a - \frac{b}{c})t = a(1 - \frac{v}{c})t$$

$$ct = (ac+b)t' = (ac+b)a(1-\frac{v}{c})t = a^2(c+v)(1-\frac{v}{c})t$$

$$= a^2c(1+\frac{v}{c})(1-\frac{v}{c})t = a^2ct\left(1-\left(\frac{v}{c}\right)^2\right)$$

$$\Rightarrow a^2 = \frac{1}{1-(v/c)^2} \Rightarrow \boxed{a = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

$$\left.\begin{aligned}
\Rightarrow x &= \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}(x' + vt') = \gamma(x' + vt') \\
x' &= \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}(x - vt) = \gamma(x - vt) \\
&\text{donde } \gamma(v) \equiv \left(1-(v/c)^2\right)^{-1/2}
\end{aligned}\right\} \text{Transf. de Lorentz}$$



<u>Ejercicio</u>: Dadas las T. de L, obtenga

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

Para las demás coordenadas, $y' = y$

$z' = z$

T. de los tiempos

$$x = \gamma(x' + vt')$$

$$\Rightarrow vt' = x\left(\frac{1}{\gamma} - \gamma\right) + \gamma v t$$

pero, $\frac{1}{\gamma} - \gamma = \sqrt{1-\beta^2} - \frac{1}{\sqrt{1-\beta^2}}$ $\quad (\beta \equiv v/c)$

$$= \frac{-\beta^2}{\sqrt{1-\beta^2}} = -\beta^2\gamma$$

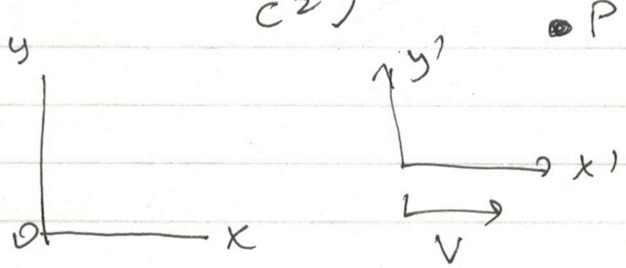
$$\Rightarrow vt' = \gamma v t - \beta^2 \gamma x = \gamma(vt - \beta^2 x)$$

$$t' = \gamma\left(t - \frac{1}{v}\left(\frac{v}{c}\right)^2 x\right)$$

$$\therefore \boxed{t' = \gamma\left(t - \frac{vx}{c^2}\right)}$$

Transf. de velocidade

$$x = \gamma(x' + vt') \quad ; \quad y = y'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$



em S': $\quad u_x' = \dfrac{dx'}{dt'} \quad ; \quad u_y' = \dfrac{dy'}{dt'}$

$$dx = \gamma(dx' + v\,dt')$$
$$dt = \gamma\left(dt' + \frac{v}{c^2}dx'\right)$$

$$\Rightarrow u_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'} = \frac{\cancel{dt'}(u_x' + v)}{\cancel{dt'}\left(1 + \frac{v u_x'}{c^2}\right)}$$

$$\therefore \boxed{u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}} \Rightarrow \boxed{u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{\gamma\left(dt' + \frac{v}{c^2}dx'\right)} = \boxed{\frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}}$$

$$\Rightarrow \boxed{u_y' = \frac{u_y/\gamma}{1 - \frac{v u_x}{c^2}}}$$

igual para u_z'

EX: $u_x' = v = 0.5c$

$$u_x = \frac{0.5c + 0.5c}{1 + (0.5)^2} = \frac{4}{5}c$$

En gral, si $v = \beta_1 c$ y $u_x' = \beta_2 c$ $(0 < \beta_1, \beta_2 < 1)$

$$\Rightarrow \frac{u_x}{c} = \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$$

$$\Rightarrow 1 - \beta = 1 - \frac{(\beta_1 + \beta_2)}{1 + \beta_1\beta_2} = \frac{1 + \beta_1\beta_2 - \beta_1 - \beta_2}{1 + \beta_1\beta_2} = \frac{(1-\beta_1)(1-\beta_2)}{1 + \beta_1\beta_2}$$

o sea si $0 < \beta_1 < 1$ y $0 < \beta_2 < 1 \Rightarrow 0 < 1 - \beta < 1$

siempre.

~~[illegible]~~

En (S) se obs. q' un evento tomo lugar en A sobre eje x y 10^{-6}s después otro evento ocurre en B, loc. a 900 m de A.
Hallar magnitud y dirección de (S') c/r a (S) tq. ambos eventos aparezcan simultáneos.

A: $t_A = 0$, $x_A = 0$
B: $t_B = 10^{-6}$ s, $x_B = 900$ m

$$t_A' = \gamma \left(t_A - \frac{v x_A}{c^2}\right)$$

$$t_B' = \gamma \left(t_B - \frac{v x_B}{c^2}\right)$$

$$\underbrace{t_B' - t_A'}_{0} = \gamma (t_B - t_A) - \frac{\gamma v}{c^2}(x_B - x_A)$$

$$t_B - t_A = \frac{v}{c^2}(x_B - x_A) \Rightarrow \frac{v}{c} = \frac{c(t_B - t_A)}{x_B - x_A} = \frac{1}{3}$$

Eventos

Evento 1: $x_1' = \gamma(x_1 - v t_1)$ $\qquad$ $x_1 = \gamma(x_1' + v t_1')$

$t_1' = \gamma\left(t_1 - \frac{v}{c^2} x_1\right)$ $\qquad$ $t_1 = \gamma\left(t_1' + \frac{v}{c^2} x_1'\right)$

Evento 2: $x_2' = \gamma(x_2 - v t_2)$ $\qquad$ $x_2 = \gamma(x_2' + v t_2')$

$t_2' = \gamma\left(t_2 - \frac{v}{c^2} x_2\right)$ $\qquad$ $t_2 = \gamma\left(t_2' + \frac{v}{c^2} x_2'\right)$

luego,

$$x_2' - x_1' = \gamma\left[(x_2 - x_1) - v(t_2 - t_1)\right]$$
$$t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right]$$

EX: sist. S' tiene veloc. v $\underset{= 0'6c}{}$ c/r a S. Se ajustan los relojes t.q. $t = t' = 0$ cuando $x = x' = 0$.

evento 1 ocurre en $x_1 = 10\,m$, $t_1 = 2 \times 10^{-7}$ secs ($y_1 = 0 = z_1$)

evento 2 ocurre en $x_2 = 50\,m$, $t_2 = 3 \times 10^{-7}$ secs, ($y_2 = 0 = z_2$)

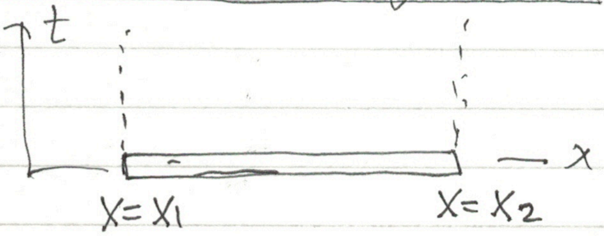
¿cuál es la distancia entre ambos eventos, medidos en S'?

$(v/c)^2 = (9/25) \Rightarrow \gamma = (1 - (v/c)^2)^{-1/2} = 5/4$

$\Rightarrow x_2' - x_1' = \frac{5}{4}\left[(50-10) - \left(\frac{3}{5}\right) 3 \times 10^8 (3-2) 10^{-7}\right] = 27.5\,(m).$

Contracción de longitudes

sist. S:



$l = x_2 - x_1$ (en el sist. en reposo)

¿cuál es la long. de la barra ~~...~~ medida en $\underline{\underline{\text{otro}}}$ sist. S'?

$\rightarrow$ medir las posiciones de ambos extremos (x_1' y x_2') al mismo tiempo t' (medido en S')

$$l' = x_2' - x_1'$$

sabemos
$$x_1 = \gamma(x_1' + vt')$$
$$x_2 = \gamma(x_2' + vt')$$

$$\Rightarrow \underbrace{x_2 - x_1}_{l} = \gamma(\underbrace{x_2' - x_1'}_{l'}) \Rightarrow l' = \frac{l}{\gamma} = l\sqrt{1-\left(\frac{v}{c}\right)^2} < l$$

<u>Dilatación del tiempo</u>: Sup. un reloj en reposo en $x = x_0$ en S.

evento 1: (x_0, t_1) "TIC"
evento 2: (x_0, t_2) "TAC"

¿Cómo se ve esto desde S'?

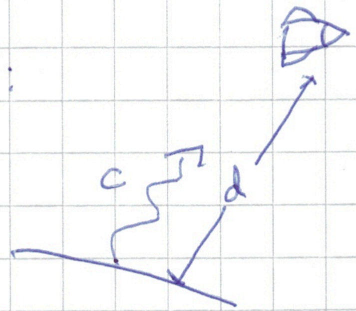
$$\left.\begin{aligned} t_1' &= \gamma\left(t_1 - \frac{vx_0}{c^2}\right) \\ t_2' &= \gamma\left(t_2 - \frac{vx_0}{c^2}\right) \end{aligned}\right\} \Rightarrow \underbrace{t_2' - t_1'}_{} = \gamma(t_2 - t_1)$$

$$\Delta\tau' = \gamma\,\Delta\tau > \Delta\tau$$

$\Rightarrow$ reloj parece correr "más lento" al ser observado desde S'.

una nave se aleja de la T a $v = 0.8c$. cuando se encuentra a una distancia $d = 6{,}66 \times 10^{8}$ km, se le envía una señal de radio desde la T ¿cuánto tarda en llegar la señal, medido en ambos sist. de referencia? ¿cuál es la posición de la nave cuando recibe la señal, en ambos sist. de referencia?

sistema tierra (S):



$$ct = d + vt$$

$$(c-v)t = d$$

$$\boxed{t = \frac{d}{c-v}}$$

y la posición será $ct = \boxed{\dfrac{dc}{c-v}} = x$

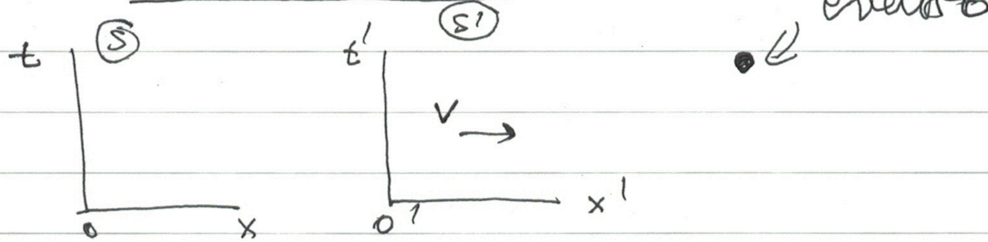
sistema nave (S'): transformamos $\underbrace{(x, t)}_{\text{evento}} \rightarrow (x', t')$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \gamma\left[\frac{d}{c-v} - \frac{v \cdot dc}{c^2(c-v)}\right] = \frac{\gamma}{c-v}\left(d - \frac{v}{c}d\right)$$

$$= \frac{\gamma d}{c-v}\left(1 - \frac{v}{c}\right) = \frac{\gamma d}{c\cancel{(1-\frac{v}{c})}}\cancel{\left(1-\frac{v}{c}\right)} = \boxed{\frac{\gamma d}{c}}$$

$$x' = 0$$

Invariante



$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$

Evaluemos: $(ct')^2 - (x')^2$

$$= \gamma^2\left(ct - \frac{vx}{c}\right)^2 - \gamma^2(x - vt)^2$$

$$= \gamma^2\left[c^2t^2 + \frac{v^2x^2}{c^2} - 2vxt\right] - \gamma^2\left[x^2 + v^2t^2 - 2xvt\right]$$

$$= \gamma^2\left[c^2t^2 + \frac{v^2x^2}{c^2} \cancel{- 2vxt} - x^2 - v^2t^2 \cancel{+ 2vxt}\right]$$

$$= \gamma^2\left[(c^2 - v^2)t^2 - x^2\left(1 - \left(\frac{v}{c}\right)^2\right)\right] = \gamma^2\left[c^2\left(1 - \left(\frac{v}{c}\right)^2\right)t^2 - x^2\left(1 - \left(\frac{v}{c}\right)^2\right)\right]$$

$$= \cancel{\gamma^2\left(1 - \left(\frac{v}{c}\right)^2\right)}\left[(ct)^2 - x^2\right] = (ct)^2 - x^2$$

$$\therefore \quad s^2 \equiv (ct')^2 - (x')^2 = (ct)^2 - x^2$$

INVARIANTE RELATIVISTA.

también: $E_0^2 \equiv E^2 - c^2p^2$

E_0: reposo

E: total

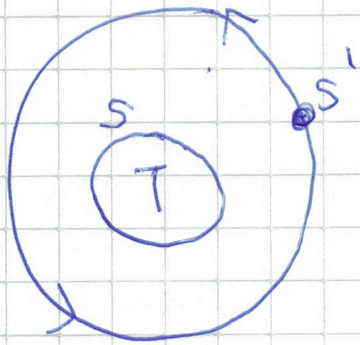
An observer on Earth observes two spacecraft moving in the *same* direction toward the Earth. Spacecraft A appears to have a speed of $0.50c$, and spacecraft B appears to have a speed of $0.80c$. What is the speed of spacecraft A measured by an observer in spacecraft B?

En gral, $\vec{v}_A = \dfrac{\vec{v}_B + \vec{v}_A{}'}{1 + \dfrac{\vec{v}_A{}' \cdot \vec{v}_B}{c^2}} \Rightarrow$

$\vec{v}_A{}' = \dfrac{\vec{v}_A - \vec{v}_B}{1 - \dfrac{\vec{v}_A \cdot \vec{v}_B}{c^2}}$. In our case $\vec{v}_A = -0.5c\,\hat{i}$
$\vec{v}_B = -0.8c\,\hat{i}$

$\Rightarrow v_A{}' = \dfrac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$

In 1962, when Scott Carpenter orbited Earth 22 times, the press stated that for each orbit he aged 2 millionths of a second less than if he had remained on Earth. (a) Assuming that he was 160 km above Earth in an eastbound circular orbit, determine the time difference between someone on Earth and the orbiting astronaut for the 22 orbits. (b) Did the press report accurate information? Explain.



$$\Delta\tau = \tau - \tau' = \tau - (1/\gamma)\tau = \left(1 - \frac{1}{\gamma}\right)\tau$$

$$\tau \approx \frac{2\pi R}{v} = 2\pi\sqrt{R/g} \qquad (1)$$

$$\frac{GmM}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad (0)$$

$$\gamma = \left(1 - (v/c)^2\right)^{-1/2} \simeq 1 + \frac{1}{2}(v/c)^2$$

$$\Rightarrow \frac{1}{\gamma} \simeq 1 - \frac{1}{2}(v/c)^2 \Rightarrow 1 - \frac{1}{\gamma} = \frac{1}{2}(v/c)^2$$

$$\Rightarrow \Delta\tau = \cancel{2}\pi\sqrt{R/g}\,\frac{1}{\cancel{2}}\left(\frac{v}{c}\right)^2 \overset{(0)}{=} \frac{\pi R^{3/2} g^{1/2}}{c^2} \qquad (2)$$

$R = 6400$ km
$g = 9.8$ m/s

$$\Rightarrow \boxed{\Delta\tau \approx 1.78\,\mu s}$$