

Familia cuadrática

martes, 6 de octubre de 2020 16:31

A continuación veremos la dinámica de una familia de aplicación. y estudiaremos sus bifurcaciones.

~~Ex~~ ① $Q_c(x) = x^2 + c$, c es d

Pts fijos de la familia : $Q_c(x)$

$$x^2 + c = x$$

$$x^2 - x + c = 0$$

$$x = \frac{1 \pm \sqrt{1-4c}}{2}$$

$$x = \frac{1 + \sqrt{1-4c}}{2}$$

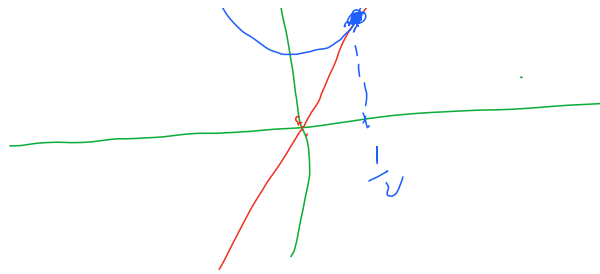
$$x = \frac{1 - \sqrt{1-4c}}{2}$$

Veremos a qué da $1-4c$ para ver si hay bifurcaciones.

Caso 1 $1-4c = 0 \Rightarrow c = \frac{1}{4}$

|||

$$\Rightarrow x = \frac{1}{2}$$



Case 2

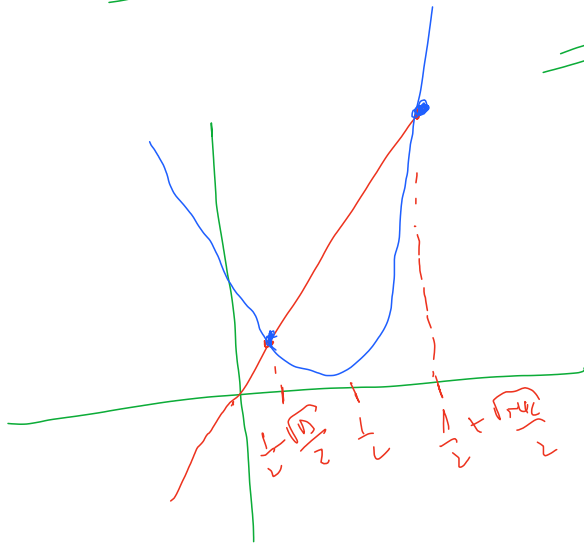
$$1 - 4C > 0$$

$$\Rightarrow D \subset \subset \frac{1}{2} \mathbb{Z}$$

→ 2 pt.
Hlea

$$x = \frac{1}{2}$$

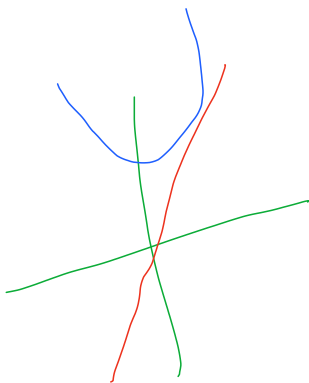
$$x = \frac{1}{2}$$



Woz 3

1-4C2D

$$\Rightarrow C \geq \frac{1}{2}$$



$\Rightarrow$ No way positive
reales

Obs: Debi do a que para dife
valores de c , combinou ha
bilidade de Diferença

Com tñdad de μ no tro μ no dice que existen bñ funcões

$$(2) \quad \bar{F}_{\mu}(x) = \mu x(1-x), \quad \mu > 0$$

pts fijos: $\bar{F}_{\mu}(x) = x$

$$\mu x(1-x) = x$$

$$\mu x - \mu x^2 - x = 0$$

$$x(\mu - 1 - \mu x) = 0$$

$$x = 0$$

$$x_{\mu} = \frac{\mu - 1}{\mu}$$

pts fijos

$$|f'(p)|$$

Naturaleza de los fijos (Atracción o Repulsión)

$$f'_{\mu}(x) = \mu - 2\mu x$$

$$0 < \mu < 1 \quad 1 < \mu < 2 \quad \mu > 2$$

$$1^{\text{to}} \text{ PV } \boxed{X=0} \quad | | f_{\mu}(0) | = |\mu| = \mu.$$

$\Rightarrow$ $X=0$ is repulsion

pt of fixo $x_{\mu} = \frac{\mu-1}{\mu}$

$$\begin{aligned} | f'_{\mu}(\frac{\mu-1}{\mu}) | &= | \mu - 2\mu \cdot \frac{\mu-1}{\mu} | \\ &= | \mu - 2\mu + 2 | \\ &= | 2 - \mu | \end{aligned}$$

Segun sea μ , x_{μ} sea repul
atractor o node.

x_{μ} sea atractor

$$| f'_{\mu}(x_{\mu}) | < 1$$

$$| 2 - \mu | < 1$$

$$-1 < 2 - \mu < 1$$

$$-3 < -\mu < -1$$

$$1 < \mu < 3$$

Para $\mu \in (1, 3) \Rightarrow x_\mu = \frac{\mu-1}{\mu}$ e

x_μ repulsor $|f'_\mu(x_\mu)| >$

$$|2-\mu| > 1$$

$$2-\mu > 1$$

0

$$2-\mu < -1$$

$$(1 > \mu)$$

$\Rightarrow$

$$\boxed{\mu > 3}$$

Para $\mu > 3$

Para $\mu > 3 \Rightarrow x_\mu$ e repulsor

obs 11

$$S: \mu = 3$$

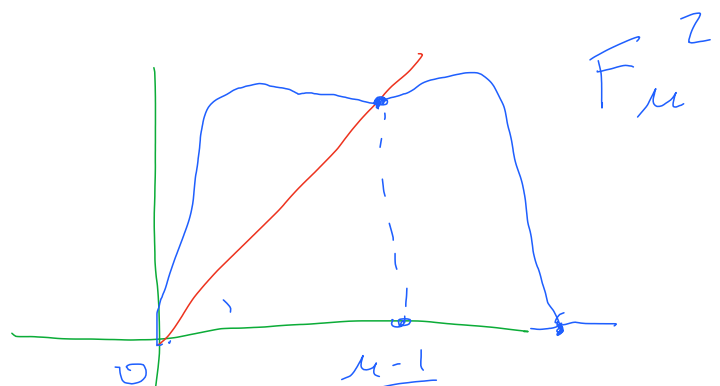
$$1 \leq \mu < 3 \quad |2-\mu| =$$

$||u' \times u|| = 1$
 ahí no sabemos quié es.

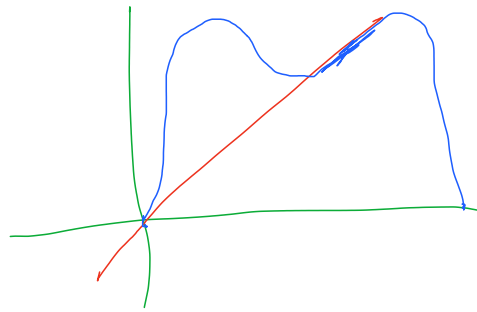
obs 1 Pese a que si μ cambia la cantidad de pts fijos se mantiene, la naturaleza del pto fijo es la que cambia. esto también se le llama bifurcación.

obs 2: El libro también estudió por medio de gráficos los pts fijos para $F_\mu^2(u)$, esto es los pts periódicos de periodo 2 de F_μ .

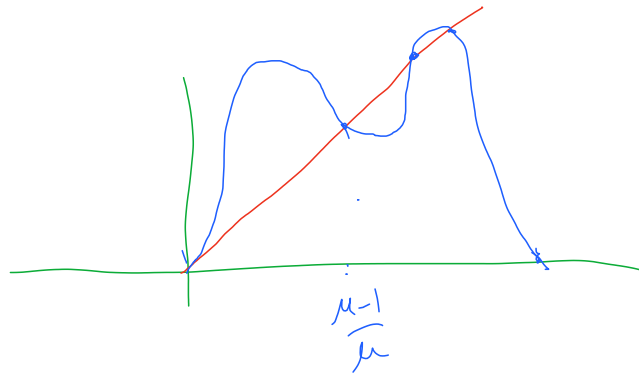
$$1 < \mu < 3$$



$$\mu = 3$$



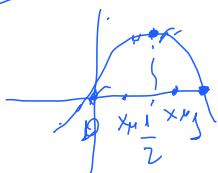
$$\mu > 3$$



$$F_\mu = \mu x(1-x)$$

Debido a que la familia Cuadrática muestra fenómenos de muchos sistemas dinámicos interesantes, vamos a continuar estudiando de ella.

Prop (1)



i) $F_\mu(0) = 0 = F_\mu(1)$

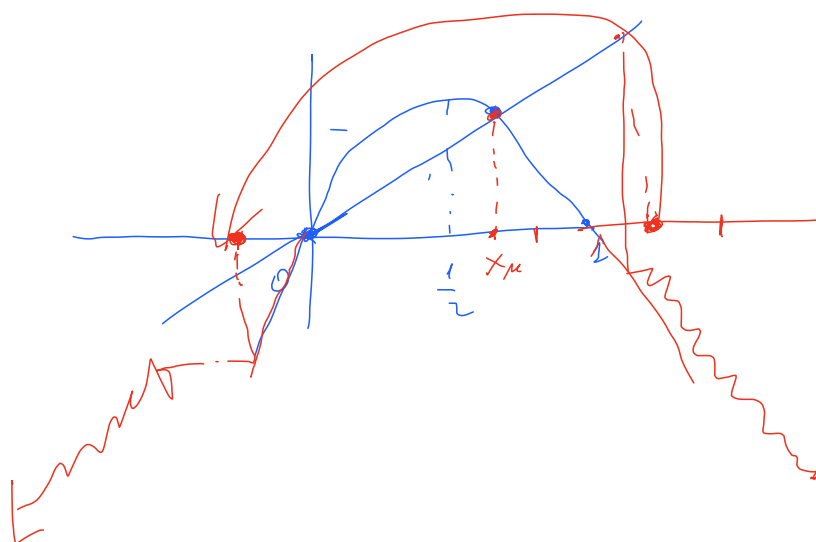
ii) si $\mu > 1$

$$\Rightarrow 0 < X_\mu < 1$$

Prop (2) Suponga que $\mu > 1$

$$\text{Si } x < 0 \Rightarrow F_\mu^n(x) \xrightarrow{n \rightarrow \infty}$$

$$\text{Si } x > 1 \Rightarrow F_\mu^n(x) \xrightarrow{n \rightarrow \infty}$$



Dem 1 Si $x < 0 \Rightarrow \mu x(1-x) <$

$$\Rightarrow \bar{F}_\mu(x) < x$$

$$\Rightarrow \bar{F}_\mu^n(x) < \bar{F}_\mu^{n-1}(x) < \dots < \bar{F}_\mu(x) < x$$

$\bar{F}_\mu^n$ decrece. $\bar{F}_\mu^n(x) = \bar{F}_\mu(x) < \bar{x}$

$a \rightarrow \infty$ ✓

✓

en un pto x
 P

$$\lim_{n \rightarrow \infty} \bar{F}_\mu^n(x) = P$$

$$\lim_{n \rightarrow \infty} F_\mu^{n'}(x) = \bar{F}_\mu$$

~~X~~

$$\text{Si } X > 1 \Rightarrow \text{como } \bar{F}_\mu(1) =$$

$$\Rightarrow \bar{F}_\mu(X) < \infty$$

$$\Rightarrow \bar{F}_\mu^n(x) \xrightarrow{n \rightarrow \infty} \infty$$

Prop (3) Sea $1 < \mu < 3$, entonces

(i) $\bar{F}_\mu$ tiene pto fijo atractivo
 en $X_\mu = \frac{\mu-1}{\mu}$ y repulsor
 $x=0$. ✓

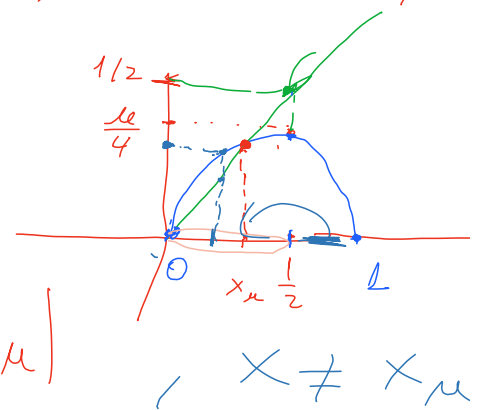
(ii) Si $0 < X < 1$

(2)

$$\lim_{n \rightarrow \infty} F_\mu^n(x) = x_\mu.$$

Dem (2i) Para demostrar esto
vamos a separar $\mu \in (1, 3)$.

Si $\mu \in (1, 2)$ | $F_\mu(x) = \mu x(1-x)$
y $x \in (0, \frac{1}{2}]$



$$|F_\mu(x) - x_\mu| < |x - x_\mu|, \quad x \neq x_\mu$$

$$\Rightarrow |F_\mu(x) - x_\mu| \leq \lambda |x - x_\mu|, \quad \text{donde } \lambda < 1$$

$$\begin{aligned} \Rightarrow |F_\mu^n(x) - x_\mu| &\leq \lambda |F_\mu^{n-1}(x) - x_\mu| \\ &\leq \lambda^n |x - x_\mu| \end{aligned}$$



$$\left(\lim_{n \rightarrow \infty} T_n(x) = x_\mu \right)$$

$$S; \mu < 2$$

$$S; x > \frac{1}{2}$$

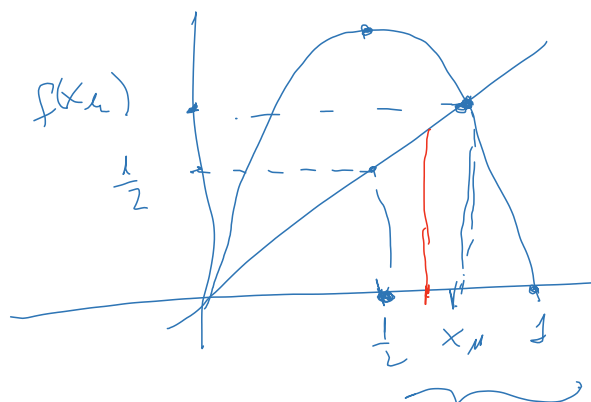
$$F_n\left(\frac{1}{2}, 1\right) \subset (0, 1)$$

$$\lim_{n \rightarrow \infty} T_n^n(x) = x_\mu$$

$$2 < \mu < 3$$

observe graph

$$\frac{1}{2} < x_\mu < 1$$



denote $\hat{x}_\mu = f(x_\mu)$

$$\Rightarrow F_\mu^n(\hat{x}_\mu) \rightarrow x_\mu$$

$$\Rightarrow F_\mu^{n+1}(x_\mu) \rightarrow x_\mu$$

~~Answer~~

x_μ

$\mu \in (1, 2)$

$\mu \in (2, 3)$



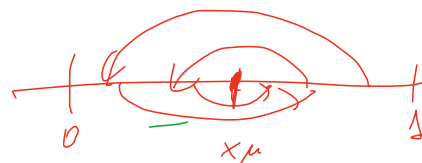
$$S; x < x_\mu \Rightarrow \exists K \perp_p$$

+

$\Omega_i \quad \mu \in (1, 2) \quad X_\mu$ autre $\mathcal{M}$

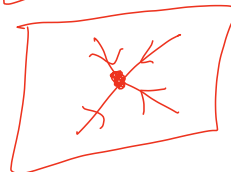
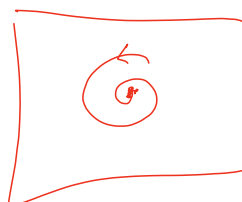
A hand-drawn diagram on a white background. It features a horizontal black line. Above this line, there is a green wavy line. A red arc is drawn above the green wavy line, spanning from the left edge to the right edge. A red dot is placed on the horizontal line, slightly to the right of the center. Below the horizontal line, there are several handwritten marks: a 'u' on the left, and 'X' and '1' near the red dot. On the right side of the horizontal line, there are two vertical tick marks.

Matrix of $\mathcal{C}^1$



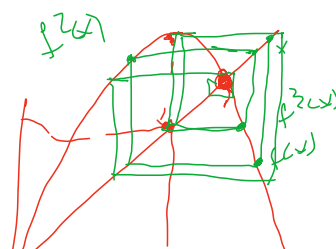
obs 1 Δ esto toubien se le
dans bifurcation.

Sist.



C¹ no parido

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$$2 \leq \mu \leq 3$$




$$\#_{\mu}^n = \frac{1}{d} \frac{\phi}{\mu}$$

Tercera lista evaluada.

La Aiperbolicidad Reg 30-31
Deveney