P1)

Considere la función $F:[1,18]\to\mathbb{R}$ por

$$F(t)=\int_t^{2t}\int_x^{x+1}\left(2u-\frac{1}{u^2}\right)du\,dx$$

Determine $F'(t)$.

Solución: sea $g(x)=\int_x^{x+1} 2u-\frac{1}{u^2}\,du$, de modo que

$$F(t)=\int_t^{2t} g(x)\,dx=\int_t^{1} g(x)\,dx+\int_1^{2t} g(x)\,dx$$

$$=\int_1^{2t} g(x)\,dx-\int_1^{t} g(x)\,dx \quad \text{y usando TFC,}$$

$$F'(t)=\left[\int_1^{2t} g(x)\,dx\right]'-\left[\int_1^{t} g(x)\,dx\right]'$$

$$=g(2t)\cdot[2t]'-g(t)=2g(2t)-g(t).$$

En que $g(x)=\int_x^{x+1} 2u-\frac{1}{u^2}\,du=\int_x^{x+1} 2u\,du+\int_x^{x+1}\frac{-1}{u^2}\,du$

como $[u^2]'=2u$ y $[1/u]'=\frac{-1}{u^2}$, obtenemos,

$$g(x) = \int_x^{x+1} 2u\,du + \int_x^{x+1} \frac{-1}{u^2}\,du$$

$$= u^2\Big|_x^{x+1} + \frac{1}{u}\Big|_x^{x+1} = (x+1)^2 - x^2 + \frac{1}{x+1} - \frac{1}{x}$$

$$= 2x+1 + \frac{x-(x+1)}{x(x+1)} = 2x+1 - \frac{1}{x(x+1)}$$

$$\therefore\ g(x) = 2x+1 - \frac{1}{x(x+1)}$$

luego,

$$F'(t) = 2\,g(2t) - g(t) = 2\left(2(2t)+1 - \frac{1}{(2t)(2t+1)}\right)$$

$$- \left(2t+1 - \frac{1}{t(t+1)}\right)$$

concluimos,

mala mía !!

$$\cancel{F'(t) = 2\left(4t+1 - \frac{1}{2t(2t+1)} - 2t - 1 + \frac{1}{t(t+1)}\right)}$$

$$F'(t) = 2\left(4t+1 - \frac{1}{2t(2t+1)}\right) - \left(2t+1 - \frac{1}{t(t+1)}\right)$$