Taller 2

1

P1 $f: \mathbb{R} \to \mathbb{R}$ por $f(x) = (1+x)^{100}$

a) Recta tangente en $(0, f(0))$

vemos que $f(0) = 1^{100} = 1$ y la ec de la recta tangente viene dada por:

$$y = f(0) + f'(0)(x-0)$$

$$= 1 + f'(0) \cdot x$$

Encontremos $f'(0)$

$$f(x) = (1+x)^{100} \to f'(x) = 100(1+x)^{99} \cdot [1+x]'$$

$$= 100(1+x)^{99} \cdot 1 = 100(1+x)^{99}$$

$$\Rightarrow f'(0) = 100(1+0)^{99} = 100$$

luego,

$$y = 1 + 100x \text{ //}$$

b) Estime $f(\frac{1}{1000})$

Reemplazando en la recta tangente,

$$f(\tfrac{1}{1000}) \approx 1 + \frac{100}{1000} = \underline{1{,}1}$$

P2 sean f, g funciones diferenciables (2)
en $\mathbb{R}$ tales que

$$f(2)=1, \quad f'(2)=4, \quad g(2)=-5, \quad g'(2)=1$$

sea F nueva función dada por

$$F(x)=\frac{f(x)-g(x)}{4+f(x)\cdot g(x)}$$

calcule $F'(2)$

utilizando reglas de derivación,

$$F'(x) \underset{c}{=} \frac{[f(x)-g(x)]'(4+f(x)\cdot g(x))-(f(x)-g(x))[4+f(x)\cdot g(x)]'}{(4+f(x)\cdot g(x))^2}$$

En que

- $[f(x)-g(x)]' = f'(x)-g'(x)$
- $[4+f(x)\cdot g(x)]' = \underbrace{[4]'}_{\to 0} + [f(x)\cdot g(x)]'$

$$= [f(x)\cdot g(x)]' \underset{p}{=} f'(x)\,g(x) + f(x)\,g'(x)$$

Reemplazando,

3

$$F'(x) = \frac{(f'(x) - g'(x))(4 + f(x)g(x)) - (f(x) - g(x))(f'(x)g(x) + f(x)g'(x))}{(4 + f(x)g(x))^2}$$

Evaluando para $x = 2$:

$$F'(2) = \frac{(f'(2) - g'(2))(4 + f(2)g(2)) - (f(2) - g(2))(f'(2)g(2) + f(2)g'(2))}{(4 + f(2)g(2))^2}$$

$$= \frac{(4-1)(4-5) - (1+5)(4\cdot(-5)+1)}{(4-5)^2}$$

$$= \frac{-3 - 6\cdot(-19)}{1} = -3 + 114 = \underline{111}$$

P3 a) $f(x) = \sqrt[3]{\cos(1-x^2)}$ Encuentre f_1, f_2, f_3 tales que $f = f_1 \circ f_2 \circ f_3$ luego, calcule f'

Si definimos $f_3(x) = 1 - x^2$, $f_2(x) = \cos(x)$ y $f_1(x) = \sqrt[3]{x}$ vemos que

$$(f_1 \circ f_2 \circ f_3)(x) = f_1(f_2(f_3(x)))$$

$$= f_1(f_2(1-x^2)) = f_1(\cos(1-x^2))$$

$$= \sqrt[3]{\cos(1-x^2)} = f(x).$$

luego, $f'(x) = [f_1 \circ f_2 \circ f_3]'(x)$ (4)

$= [f_1 \circ (f_2 \circ f_3)]'(x) = f_1'((f_2 \circ f_3)(x)) \cdot [f_2 \circ f_3]'(x)$

$= f_1'((f_2 \circ f_3)(x)) \cdot f_2'(f_3(x)) \cdot f_3'(x)$ ☆

vemos que

- $f_3'(x) = [1 - x^2]' = [1]' - [x^2]' = 0 - 2x = 2x$
- $f_2'(x) = [\cos(x)]' = -\operatorname{sen}(x)$
- $f_1'(x) = [\sqrt[3]{x}]' = [x^{1/3}]' = \frac{1}{3} x^{-2/3}$

Así, utilizando ☆,

$f'(x) = \frac{1}{3}((f_2 \circ f_3)(x))^{-2/3} \cdot -\operatorname{sen}(f_3(x)) \cdot 2x$

$= -\frac{1}{3}(f_2(1-x^2))^{-2/3} \operatorname{sen}(1-x^2) \cdot 2x$

$= -\frac{1}{3}(\cos(1-x^2))^{-2/3} \cdot \operatorname{sen}(1-x^2) \cdot 2x$ //

b) sea f funcion diferenciable que cumple

$f(1) = 1$ / $f'(1) = 2$ sea

$G(x) = f(f((f(x))^2))$. calcule $G'(1)$

$$G'(x) = f'(f((f(x))^2)) \cdot [f((f(x))^2)]'$$

$$= f'(f((f(x))^2)) \cdot f'((f(x))^2) \cdot [(f(x))^2]'$$

$$= f'(f((f(x))^2)) \cdot f'((f(x))^2) \cdot 2f(x) \cdot f'(x)$$

Luego, $G'(1)$ queda;

$$G'(1) = f'(f(\cancelto{1}{(f(1))^2})) \cdot f'(\cancelto{1}{(f(1))^2}) \cdot 2f(\underset{1}{1}) \cdot \cancelto{2}{f'(1)} = f'(\cancelto{1}{f(1)}) \cdot f'(1) \cdot$$

$$2 \cdot 2$$

$$= \cancelto{2}{f'(1)} \cdot 2 \cdot 4 = 16.$$