P1 a) $f:[-1,6]\to\mathbb{R}$

$$f(x)=4x^2-12x+2$$

forma canónica: (vértice)

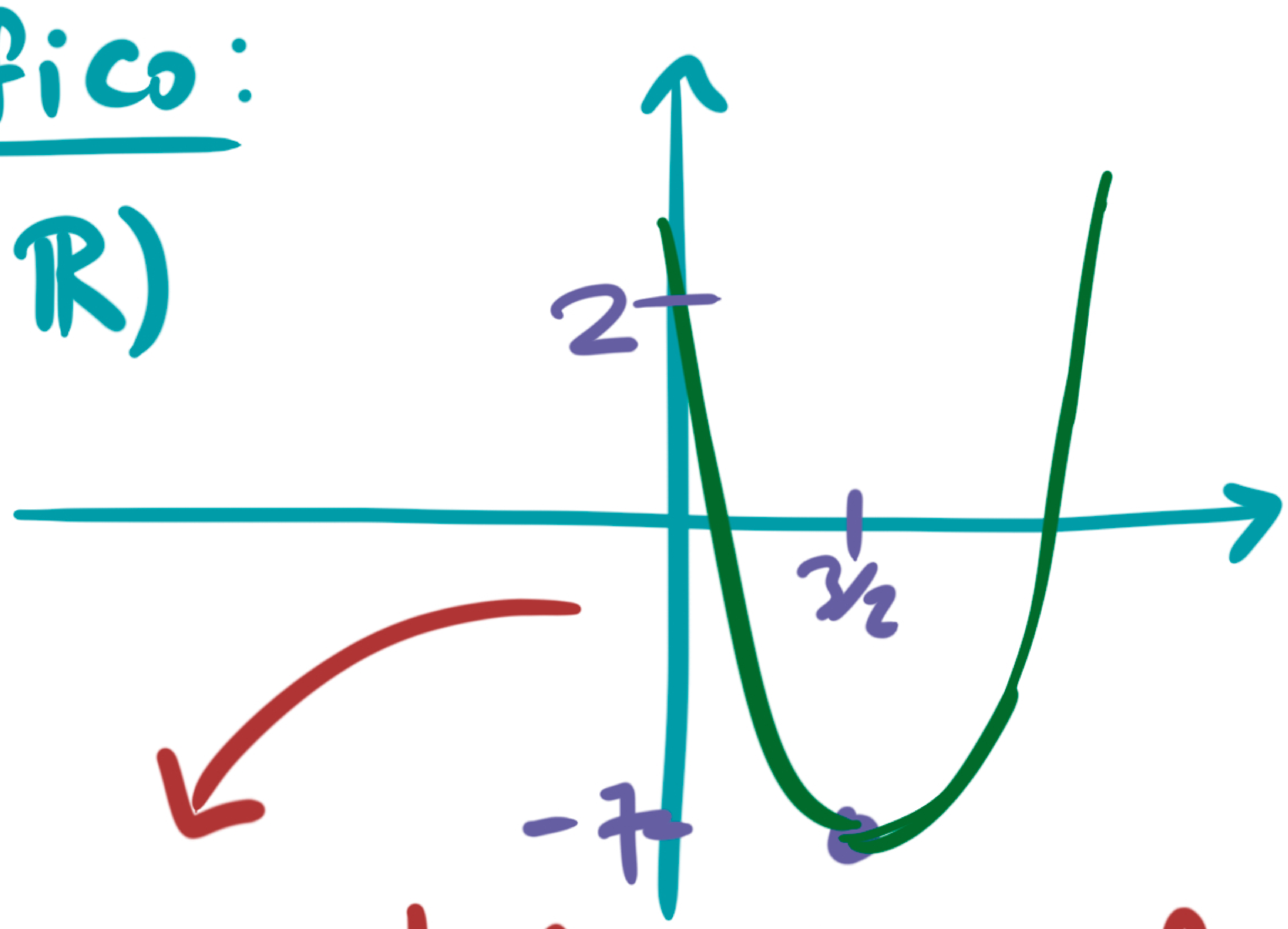
$$\frac{-b}{2a}=\frac{-(-12)}{2\cdot 4}=\frac{12}{8}=\frac{3}{2}$$

$$f\left(\frac{-b}{2a}\right)=f\left(\frac{3}{2}\right)\ 4\cdot\left(\frac{3}{2}\right)^2-12\cdot\frac{3}{2}+2$$

$$=\cancel{4}\cdot\frac{9}{\cancel{4}}-\overset{6}{\cancel{12}}\cdot\frac{3}{\cancel{2}}+2=9-18+2=-9+2$$

$$=\boxed{-7}\ \therefore f(x)=4\left(x-\frac{3}{2}\right)^2-7$$

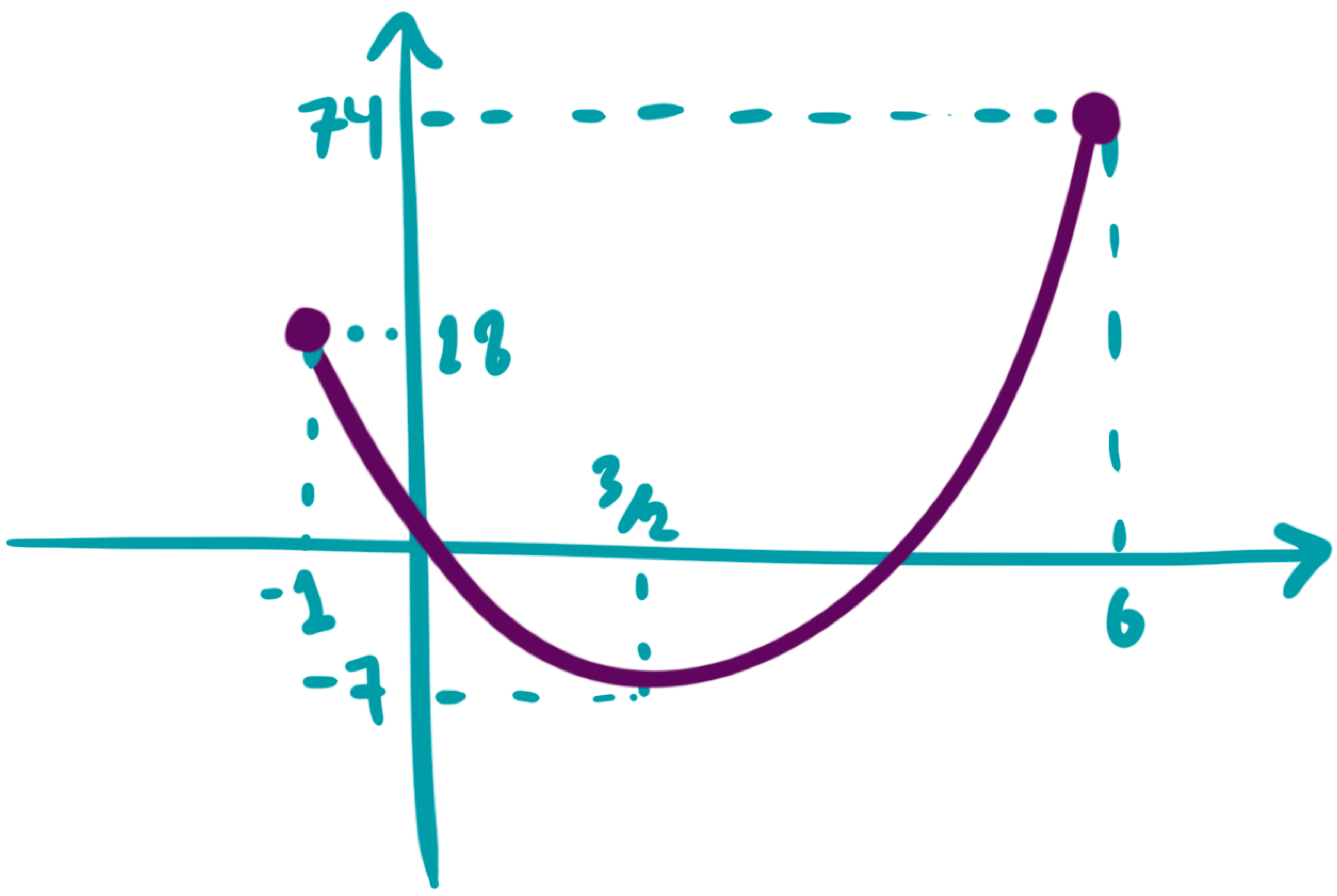
gráfico:
(en $\mathbb{R}$)



ojo: $f(0)=2$

CUIDADO! Aquí graficamos para todo $x\in\mathbb{R}$. Esto no es el gráfico de f!!

Gráfico: (en $[-1, 6]$)



nota: $f(-1) = 4\left(-1-\frac{3}{2}\right)^2 - 7$
$= \cancel{4} \cdot \frac{25}{\cancel{4}} - 7 = \underline{18}|$

$f(6) = 4\left(6 - \frac{3}{2}\right)^2 - 7$
$= 4\left(\frac{9}{2}\right)^2 - 7 = 4 \cdot \frac{81}{4} - 7$
$= \underline{74}|$

A partir del gráfico:

$\mathrm{Im}(f) = [-7, 74]$

$\therefore f$ es acotada.

b) $f : [-3, 2] \to \mathbb{R}$

$$f(x) = \frac{4x+31}{2x+12}$$

Para la forma canónica, dividimos:

$(4x+31) : (2x+12) = 2$
$-(4x+24)$
$0 + 7 /\!/$

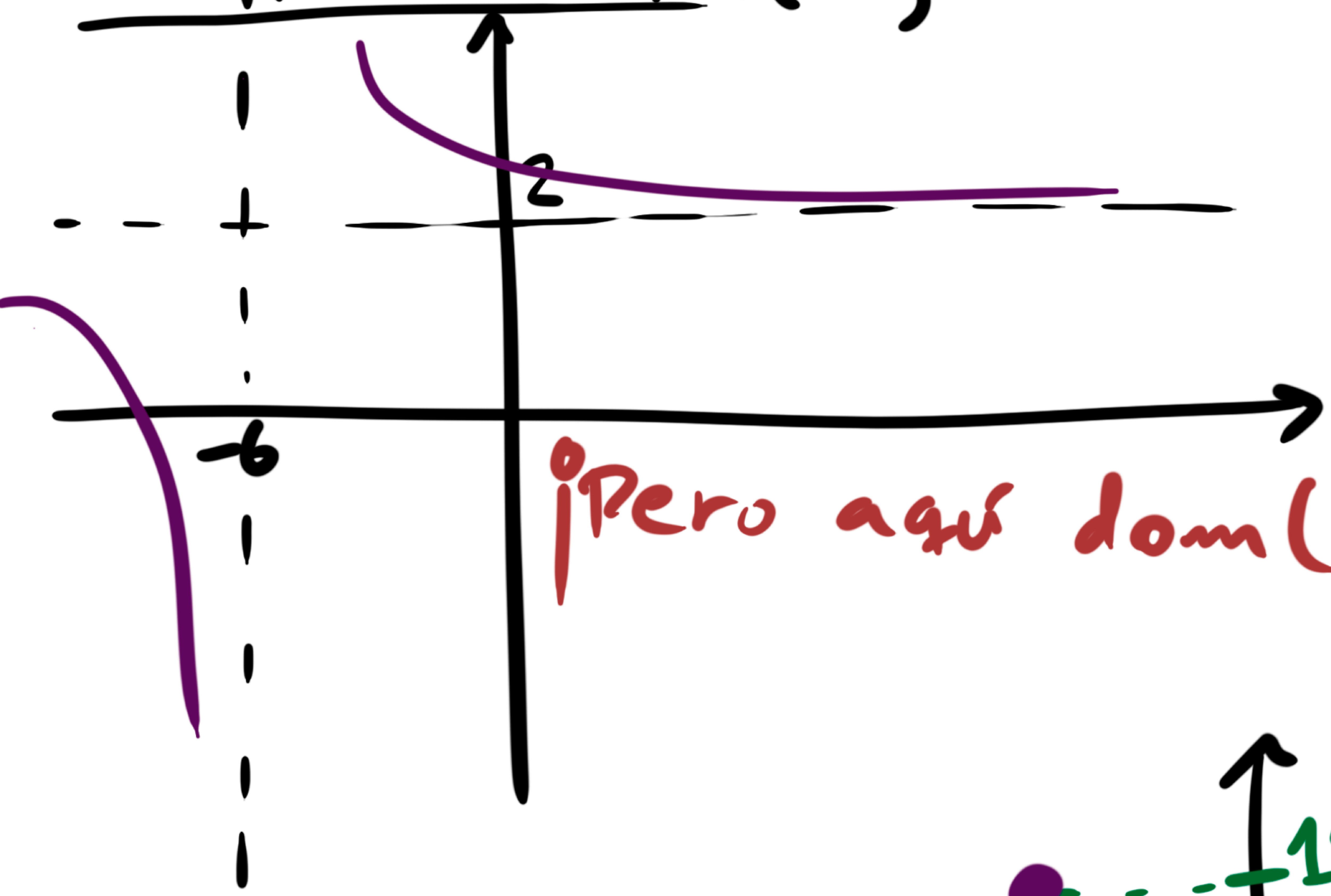
$\therefore (4x+31) = 2(2x+12) + 7$

Así:

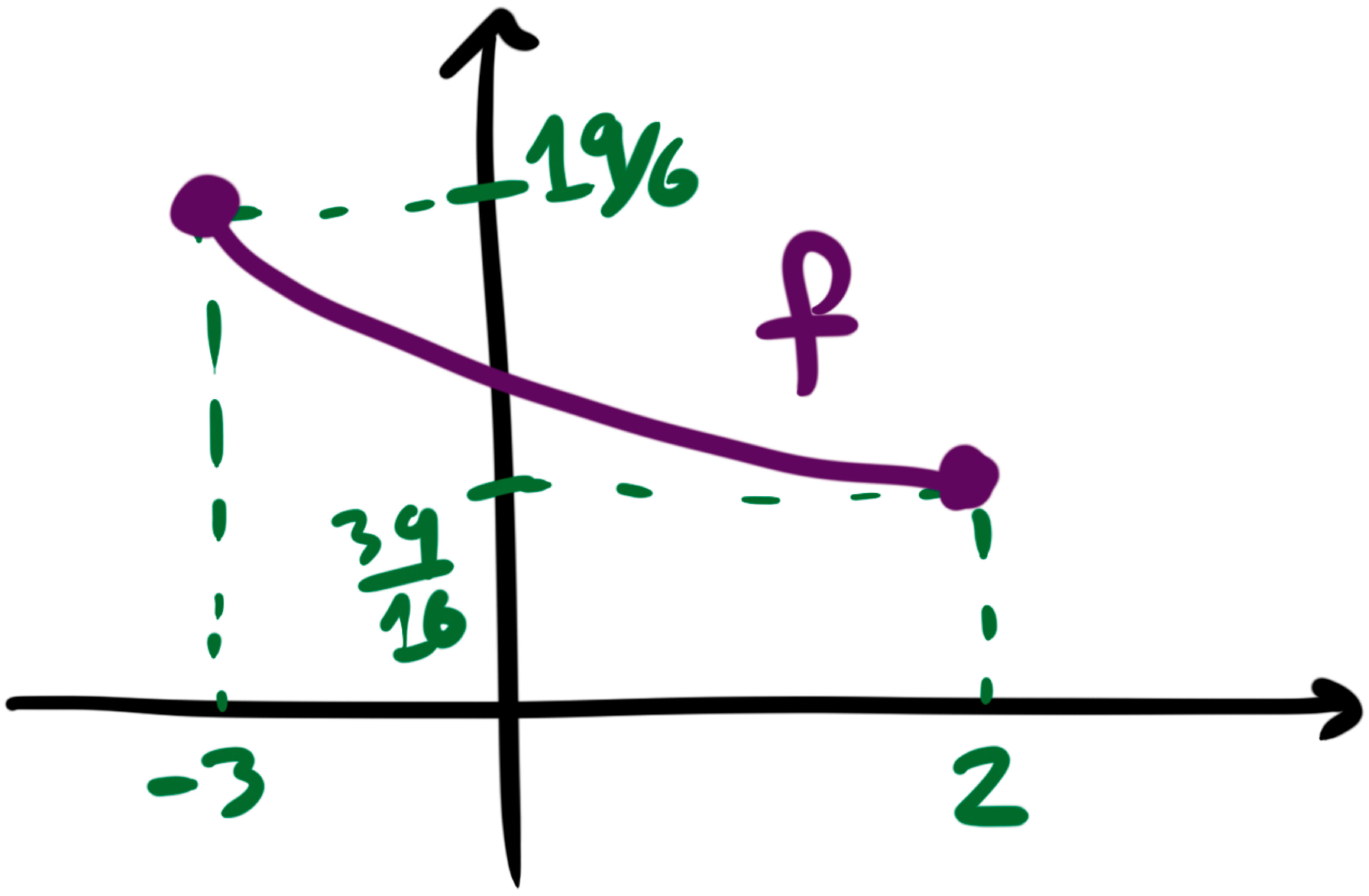
$$f(x) = \frac{2(2x+12) + 7}{2x+12} = 2 + \frac{7/2}{x+6}$$

Gráfico en $\mathbb{R}\setminus\{-6\}$

$\frac{7}{2}>0 \therefore f$ decrece.



¡Pero aquí $dom(f)=[-3,2]$!



$$f(-3)=2+\frac{7}{2}\cdot\frac{1}{-3+6}$$
$$=2+\frac{7}{2}\cdot\frac{1}{3}=\frac{12}{6}+\frac{7}{6}$$
$$=\frac{19}{6}$$

$$f(2)=2+\frac{7}{2}\cdot\frac{1}{2+6}$$
$$=2+\frac{7}{2}\cdot\frac{1}{8}=\frac{32}{16}+\frac{7}{16}$$
$$=\frac{39}{16}$$

$$Im(f)=\left[\frac{39}{16},\frac{19}{6}\right]$$

f acotada

P2) $f : [-2,2] \longrightarrow \mathbb{R}$

$f(x) = 2x+1$

$g : [-1,3] \longrightarrow \mathbb{R}$

$g(x) = x^2 - 1$

a) $\underline{f+g}$ Recuerde:

$$\operatorname{dom}(f+g) = \operatorname{dom}(f) \cap \operatorname{dom}(g)$$

$[-2,2] \cap [-1,3] = [-1,2]$. Asi:

$$f+g : [-1,2] \longrightarrow \mathbb{R}$$

$$(f+g)(x) = f(x) + g(x) = 2x+1 + x^2 - 1$$
$$= x^2 + 2x$$

$\underline{f \cdot g}$ $\operatorname{Dom}(f \cdot g) = \operatorname{dom}(f) \cap \operatorname{dom}(g)$

Luego:

$$(f \cdot g)(x) = f(x) \cdot g(x) = (2x+1)(x^2-1)$$

$\underline{f/g}$ ojo! $\operatorname{dom}(f/g) = (\operatorname{dom}(f) \cap \operatorname{dom}(g)) - \{x \in \mathbb{R} \mid g(x) = 0\}$

y también:

$$\{x\in\mathbb{R} \mid g(x)=0\}=\{x\in\mathbb{R} \mid x^2-1=0\}$$

$=\{-1,1\}$. Por lo tanto,

$$\frac{f}{g}:]-1,2]-\{1\}\longrightarrow\mathbb{R}$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2x+1}{x^2-1}$$

$f\circ g$ Recuerde que $x\in Dom(f\circ g)$ si

$$\left.\begin{array}{l} 1)\ x\in dom(g) \\ 2)\ g(x)\in dom(f) \end{array}\right\}\text{AMBOS!}$$

1) $x\in dom(g) \iff x\in[-1,3]$

2) $g(x)\in dom(f) \iff g(x)\in[-2,2]$

$$-2\leq x^2-1\leq 2 \quad / +1$$

$$\underbrace{-1\leq}_{\text{esto se cumple!}} x^2\leq 3$$

$\hookrightarrow \underset{\geq 0}{\underset{=}{1}}\,x^2-3\leq 0 \iff (x+\sqrt{3})(x-\sqrt{3})\leq 0$

$\Rightarrow x\in[-\sqrt{3},\sqrt{3}]$

Intersectamos:

$$x \in [-1,3] \cap [-\sqrt{3},\sqrt{3}] = [-1,\sqrt{3}]$$

$$(f \circ g): [-1,\sqrt{3}] \longrightarrow \mathbb{R}$$

$$(f \circ g)(x) = f(g(x)) = f(x^2-1)$$

$$= 2(x^2-1)+1 = 2x^2-1$$

Existe $f(g(3))$? No. Puesto que si bien $3 \in Dom(g)$, $g(3) = 3^2-1 = 8 \notin dom(f)$

$\underline{g \circ f}|$ Del mismo modo,

$$Dom(g \circ f) = \{x \in Dom(f) / f(x) \in dom(g)\}$$

$$= \{x \in [-2,2] / 2x+1 \in [-1,3]\}$$

$$-1 \leq 2x+1 \leq 3 \quad / -1$$

$$-2 \leq 2x \leq 2 \quad / \cdot \tfrac{1}{2} > 0$$

$$-1 \leq x \leq 1 \longrightarrow x \in [-1,1]$$

Intersectando: $x \in [-1,1]$.

Así:

$g \circ f : [-1, 1] \longrightarrow \mathbb{R}$

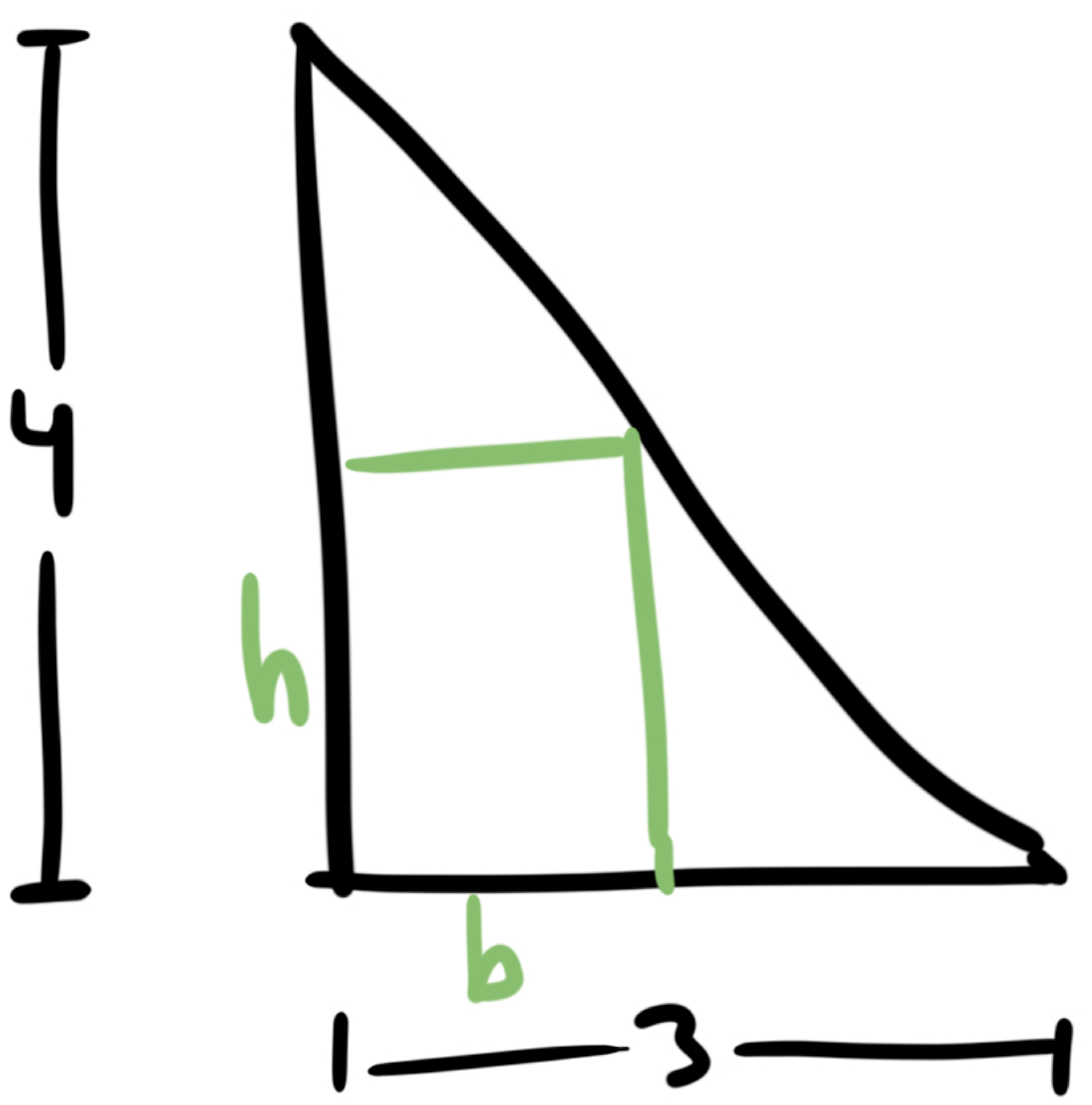
$(g \circ f)_{(x)} = g(f(x)) = g(2x+1)$

$= (2x+1)^2 - 1 = 4x^2 + 4x$

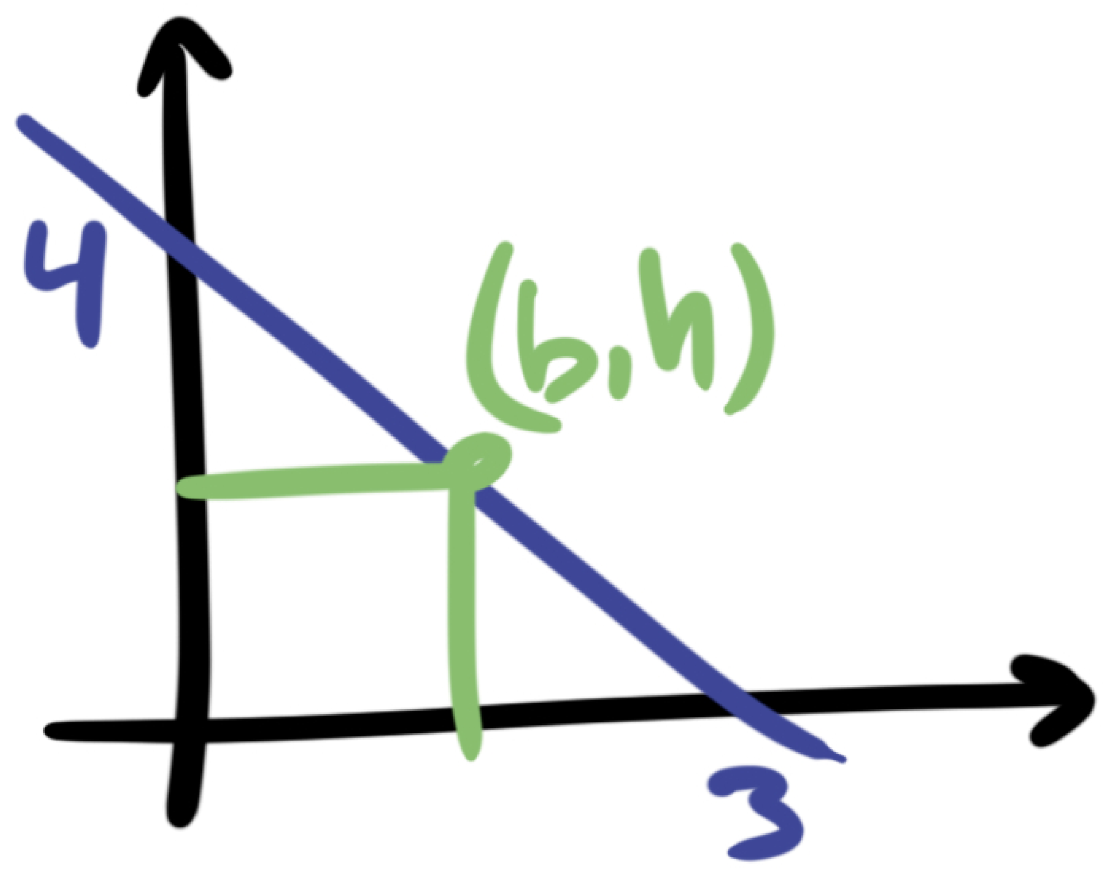
Existe $g(f(2))$? NO. Puesto que si bien $2 \in Dom(f)$, $f(2) = 2 \cdot 2 + 1 = 5 \notin Dom(g)$

Nota: $g \circ f \neq f \circ g$!

P3)



a) ubicamos un sist. de coordenadas:



(b,h) debe estar en esta recta.

pendiente: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{0-4}{3-0} = \dfrac{-4}{3}$

punto: (0,4)

$\therefore\ y = 4 - \frac{4}{3}x$

Por lo tanto:

$$h = 4 - \frac{4}{3}b \Leftrightarrow h - 4 = -\frac{4}{3}b \Leftrightarrow 4 - h = \frac{4}{3}b$$

$$\Leftrightarrow \frac{3}{4}(4-h) = b \Leftrightarrow \boxed{b = 3 - \frac{3}{4}h}$$

b) Área: $A = \frac{1}{2}bh = \frac{1}{2}\left(3 - \frac{3}{4}h\right)\cdot h$

$$= \frac{3}{2}h - \frac{3}{8}h^2$$

c) Área máxima

(como el coeficiente de h^2 es negativo, el vértice es máximo)

Vértice:

$$\frac{-b}{2a} = \frac{-\left(\frac{3}{2}\right)}{2\left(-\frac{3}{8}\right)} = \frac{-3/2}{-3/4} = \frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{\cancel{3}} = 2$$

base: $b(2) = 3 - \frac{3}{\cancel{4}_2} \cdot \cancel{2} = 3 - \frac{3}{2} = \frac{3}{2}$

Dimensiones:

$b = 3/2$ | $h = 2$

P4

$$E(x) = \frac{74x}{8x+3} \quad (x \geq 0)$$

nota: $E(0)=0$

a) forma canónica

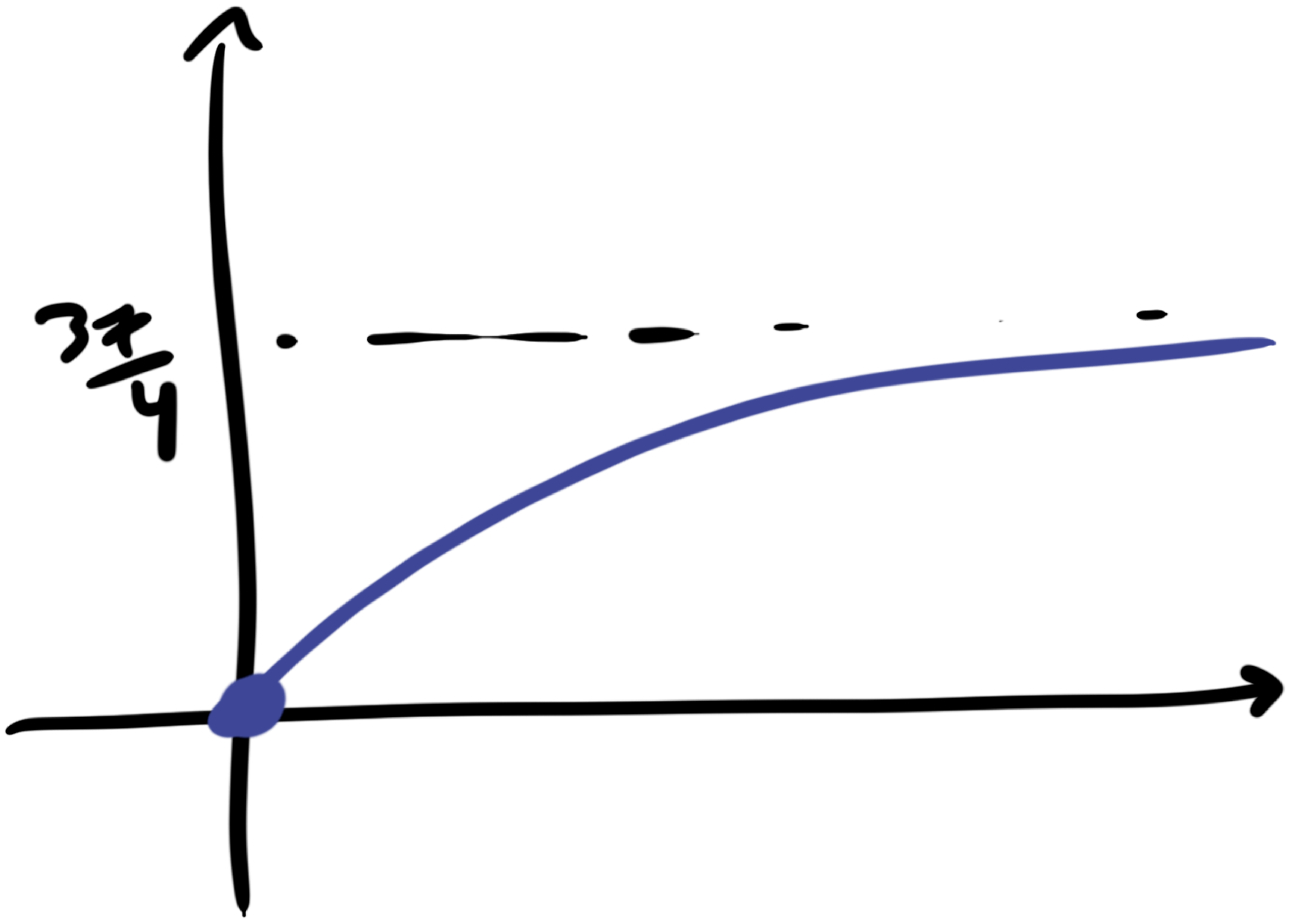
$$E(x) = \frac{74x}{8x+3} = \frac{74}{8}\cdot\frac{x}{x+3/8} = \frac{37}{4}\cdot\frac{x+\frac{3}{8}-\frac{3}{8}}{x+3/8}$$

$$= \frac{37}{4}\cdot\left(\cancelto{1}{\frac{x+\frac{3}{8}}{x+3/8}} + \frac{-3/8}{x+3/8}\right) = \frac{37}{4} - \frac{\frac{37}{4}\cdot\frac{3}{8}}{x+3/8}$$

$$= \frac{37}{4} + \frac{-111/32}{x+3/8} \to <0$$

$\therefore$ E creciente.

gráfico:



b)

$$4 \leq E(x) \leq 8$$

$$\Leftrightarrow \quad 4 \leq \frac{37}{4} - \frac{111/32}{x+3/8} \leq 8 \quad \Big/ -\frac{37}{4}$$

$$\Leftrightarrow \quad \frac{16}{4} - \frac{37}{4} \leq -\frac{111/32}{x+3/8} \leq \frac{32}{4} - \frac{37}{4}$$

$$\Leftrightarrow \quad \frac{-21}{4} \leq \frac{-111/32}{x+3/8} \leq -5/4 \quad \Big/ \cdot -1 < 0$$

$$\Leftrightarrow \quad \frac{21}{4} \geq \frac{111/32}{x+3/8} \geq 5/4 \quad \Big/ \cdot 32$$

$$\Leftrightarrow \quad 168 \geq \frac{111}{x+3/8} \geq 40 \quad \Big/ \cdot 1/111$$

$$\Leftrightarrow \quad \frac{168}{111} \geq \frac{1}{x+3/8} \geq \frac{40}{111} > 0 \quad \Big/ (\,)^{-1}$$

$$\frac{37}{56}$$

$$\Leftrightarrow \quad \frac{111}{168} \leq x + \frac{3}{8} \leq \frac{111}{40} \quad \Big/ -3/8$$

$$\Leftrightarrow \quad \frac{2}{7} \leq x \leq \frac{12}{5}$$

La dosis debe estar entre $\frac{2}{7}$ y $\frac{12}{5}$ //