

P1) Demuestre

a) Si $|x-3| < 1$, entonces

$$\frac{1}{8} < \frac{1}{x+4} < \frac{1}{6}$$

distancia

Dem: $|x-3| < 1 \Leftrightarrow 2 < x < 4$

$\Rightarrow 6 < \overset{(+4)}{x+4} < 8 \Rightarrow \frac{1}{6} > \frac{1}{x+4} > \frac{1}{8}$

$\Leftrightarrow \frac{1}{8} < \frac{1}{x+4} < \frac{1}{6} \quad \square$

b) Si $|x| < 1$, entonces

$$\sqrt{5} < \sqrt{x^2+2x+6} < 3$$

Hint:

$$x^2+2x+6 = (x+1)^2+5$$

Dem: $|x| < 1 \Rightarrow -1 < x < 1$

$\Rightarrow \underline{0} < \overset{(+1)}{x+1} < 2 \Rightarrow 0 < \overset{()^2}{(x+1)^2} < 4$

$\Rightarrow \overset{(+5)}{5} < (x+1)^2+5 < 9 \quad \sqrt{\quad} \rightarrow \text{todo es positivo!}$

$\Rightarrow \sqrt{5} < \sqrt{(x+1)^2+5} < 3 \quad \text{/hint}$

$\Rightarrow \sqrt{5} < \sqrt{x^2+2x+6} < 3 \quad \square$

P2 | Resuelva.

$$a) \frac{x+1}{x} = \frac{x+1}{x-1} - \frac{3}{x} \quad | R: \quad \underline{x \neq 1, 0}$$

$$\Leftrightarrow \frac{x+1}{x} = \frac{x+1}{x-1} \quad | \cdot x(x-1) \quad (+3/x)$$

$$\Rightarrow (x+1)(x-1) = (x+1)x$$

$$\Leftrightarrow \cancel{x^2} + 3x - 1 = \cancel{x^2} + x$$

$$\Leftrightarrow 2x = 1 \Rightarrow \underline{x = \frac{1}{2}} \quad \checkmark$$

$$b) \frac{x+6}{\sqrt{x+3}} = \sqrt{2x+12} \quad | R: \quad \begin{array}{l} \bullet x > -3 \\ \bullet x \geq -6 \end{array} \quad | ()^2$$

$$\Rightarrow \frac{x^2 + 12x + 36}{x+3} = 2x + 12 \quad | \cdot (x+3)$$

$$\Rightarrow x^2 + 12x + 36 = 2(x+6)(x+3)$$

$$\Leftrightarrow x^2 + 12x + 36 = 2(x^2 + 9x + 18)$$

$$\Leftrightarrow x^2 + 12x + \cancel{36} = 2x^2 + 18x + \cancel{36}$$

$$\Leftrightarrow 0 = x^2 + 6x$$

$$\Leftrightarrow x(x+6) = 0$$

$$1) x=0$$

$$2) x=-6 \text{ X (porque } x > -3)$$

$$\therefore \underline{x=0}$$

P3 Resuelva:

$$4x^4 - 12x^2 + 9 \leq 0 \quad \left| \begin{array}{l} \text{sea} \\ \heartsuit = x^2 \\ (\heartsuit \geq 0) \end{array} \right.$$

$$\Leftrightarrow 4\heartsuit^2 - 12\heartsuit + 9 \leq 0$$

$$\Leftrightarrow (2\heartsuit)^2 - 6 \cdot (2\heartsuit) + 9 \leq 0 \quad \left| \begin{array}{l} \text{sea} \\ \smile = 2\heartsuit \end{array} \right.$$

$$\Leftrightarrow \smile^2 - 6\smile + 9 \leq 0$$

$$\Leftrightarrow (\smile - 3)^2 \leq 0$$

$\hookrightarrow$ Solo cumple cuando $\smile = 3$

$$\Leftrightarrow 2\heartsuit = 3 \Leftrightarrow \heartsuit = \frac{3}{2} \Leftrightarrow x^2 = \frac{3}{2}$$

$$\Rightarrow x = \pm \sqrt{\frac{3}{2}}$$

P4 queremos $4 < T < 7$

con $T = \frac{76x}{8x+3}$

truco: $T = \frac{76x}{8x+3} = \frac{76}{8} \frac{8x}{8x+3}$

$$= \frac{76}{8} \left(\frac{8x+3-3}{8x+3} \right) = \frac{38}{4} \left(\frac{8x+3}{8x+3} - \frac{3}{8x+3} \right)$$

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$$= \frac{19}{2} \left(1 - \frac{3}{8x+3} \right) = \frac{19}{2} - \frac{57/2}{8x+3}$$
$$= \frac{19}{2} - \frac{57}{16x+6}$$

Ahora resolvamos la inequación!

$$4 < \frac{19}{2} - \frac{57}{16x+6} < 7 \quad | -\frac{19}{2}$$

$$\Leftrightarrow 4 - \frac{19}{2} < -\frac{57}{16x+6} < 7 - \frac{19}{2} \quad | \cdot (-1)$$

$$\Leftrightarrow \frac{19}{2} - 4 > \frac{57}{16x+6} > \frac{19}{2} - 7 \quad | \cdot \frac{1}{57}$$

$$\Leftrightarrow \frac{1}{57} \cdot \frac{11}{2} > \frac{1}{16x+6} > \frac{1}{57} \cdot \frac{5}{2} \quad | \cdot 2$$

$$\Leftrightarrow \frac{11}{57} > \frac{1}{8x+3} > \frac{5}{57} \quad | (\)^{-1}$$

$$\Leftrightarrow \frac{57}{11} < 8x+3 < \frac{57}{5} \quad | -3$$

$$\Leftrightarrow \frac{57}{11} - 3 < 8x < \frac{57}{5} - 3 \quad | \cdot \frac{1}{8}$$

$$\Leftrightarrow \frac{1}{\cancel{8}} \cdot \frac{\cancel{24}^3}{11} < x < \frac{\cancel{42}^{21}}{\cancel{5}} \cdot \frac{1}{\cancel{8}_4}$$

$$\Leftrightarrow \frac{3}{11} < x < \frac{21}{20} //$$