Taller 1

P1) a) $f(x) = \dfrac{x^2-1}{x^2+1}$

Voy a necesitar las sgtes derivadas:

i) $(x^2-1)' = 2x$

ii) $(x^2+1)' = 2x$

R. del cuociente

$\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

$\therefore f'(x) = \dfrac{2x(x^2+1) - (x^2-1)2x}{(x^2+1)^2}$

$= \dfrac{2x(\cancel{x^2}+1-(\cancel{x^2}-1))}{(x^2+1)^2} = \boxed{\dfrac{4x}{(x^2+1)^2}}$

otra forma:

$f(x) = \dfrac{x^2-1}{x^2+1} = \dfrac{x^2+1-2}{x^2+1} = \cancel{\dfrac{x^2+1}{x^2+1}}^{1} - \dfrac{2}{x^2+1}$

$= 1 - \dfrac{2}{x^2+1}$ $\therefore f'(x) = -2\left(\dfrac{1}{x^2+1}\right)'$

↳ "las ctes no cambian"

↳ esto es una composición entre $\left(\dfrac{1}{x}\right)$ y (x^2+1)

R. de la cadena:

$(f\circ g)' = (f'\circ g)\cdot g'$

$\therefore f'(x) = -2\cdot\dfrac{-1}{(x^2+1)^2}\cdot 2x = \boxed{\dfrac{4x}{(x^2+1)^2}}$

b) $g(x) = \dfrac{(x^2+x)\sqrt[3]{\operatorname{sen}(x)}}{1+x+x^2}$

derivadas:

i) $(x^2+x)' = 2x+1$

ii) $(1+x+x^2)' = 2x+1$

iii) $\left(\sqrt[3]{\operatorname{sen}(x)}\right)' = \left((\operatorname{sen}(x))^{\frac{1}{3}}\right)'$ → (cadena)

$= \frac{1}{3}\operatorname{sen}^{-\frac{2}{3}}(x)\cdot\cos(x)$

(producto) ii)

$$g'(x) = \frac{\left((x^2+x)\sqrt[3]{\operatorname{sen}(x)}\right)'(1+x+x^2) - (1+x+x^2)'(x^2+x)\sqrt[3]{\operatorname{sen}(x)}}{(1+x+x^2)^2}$$

$$= \frac{\left((2x+1)\sqrt[3]{\operatorname{sen}(x)} + (x^2+x)\frac{1}{3}\operatorname{sen}^{-\frac{2}{3}}(x)\cos(x)\right)(1+x+x^2) - (x^2+x)\sqrt[3]{\operatorname{sen}(x)}\,(2x+1)}{(1+x+x^2)^2}$$

//

c) $h(x) = \dfrac{\cos^4(x) - \operatorname{sen}^4(x)}{\cos(x)+\operatorname{sen}(x)}$ → voy a reducir esto.

$$= \frac{\cancel{(\cos^2(x)+\operatorname{sen}^2(x))}^{\,1}\,(\cos^2(x) - \operatorname{sen}(x))}{\cos(x)+\operatorname{sen}(x)} = \frac{\cancel{(\cos(x)+\operatorname{sen}(x))}(\cos(x)-\operatorname{sen}(x))}{\cancel{\cos(x)+\operatorname{sen}(x)}}$$

$= \cos(x) - \operatorname{sen}(x)$ $\therefore$ $h'(x) = (\cos(x))' - (\operatorname{sen}(x))'$

$= \underline{-\operatorname{sen}(x) - \cos(x)}$ /

d) $I(x) = \dfrac{(x^2-1)^3(x^2+1)^3}{(x^3+x^2+x+1)^3(x-1)^2} \cdot \dfrac{(x-1)}{(x-1)}$

$(x+1)\cancel{(x-1)}$

$$= \left(\frac{(x^2-1)(x^2+1)}{(x^3+x^2+x+1)\cancel{(x-1)}}\right)^3 (x-1)$$

$$= \left(\frac{(x+1)(x^2+1)}{x^3+x^2+x+1}\right)^3 (x-1) = \left(\frac{\cancel{x^3+x^2+x+1}}{\cancel{x^3+x^2+x+1}}\right)^{\cancel{3}\;1} (x-1)$$

$\therefore$ $I'(x) = 1$ //

P2 a) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1+\operatorname{sen}(3x)}{\cos(5x)}$

sean
$f(x) = 1 + \operatorname{sen}(3x)$

$g(x) = \cos(5x)$

Recordatorio: Cosas que necesitamos para usar L'Hôpital:

- f, g derivables en una vecindad de $\frac{\pi}{2}$. ✓
- $1 + \operatorname{sen}(3 \cdot \frac{\pi}{2}) = \cos(5 \cdot \frac{\pi}{2}) = 0$. ✓
- $(\cos(5x))' \neq 0$ cerca de $\frac{\pi}{2}$ ✓
- $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{(1+\operatorname{sen}(3x))'}{(\cos(5x))'}$ existe ✓

$$\lim_{x \to \frac{\pi}{2}} \frac{3\cos(3x)}{-5\operatorname{sen}(5x)} = \frac{3\overbrace{\cos\left(\frac{3\pi}{2}\right)}^{0}}{-5\underbrace{\operatorname{sen}\left(\frac{5\pi}{2}\right)}_{1}} = 0$$

Luego, por r. de L'Hôpital,

$$\lim_{x \to \frac{\pi}{2}} \frac{1+\operatorname{sen}(3x)}{\cos(5x)} = 0$$

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b) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{sen(x)} \right) = \lim_{x \to 0} \frac{sen(x) - x}{x\, sen(x)}$

$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos(x) - 1}{sen(x) + x\cos(x)} \overset{L'H}{=} \lim_{x \to 0} \frac{-sen(x)}{\cos(x) + \cos(x) - x\, sen(x)}$

$= \underline{0|}$

c) $\lim_{x \to 0} \frac{1 - \cos(x^2)}{x^4}$ | sea $u = x^2$, $x \to 0 \Rightarrow u = 0^+$

$= \lim_{u \to 0^+} \frac{1 - \cos(u)}{u^2} \overset{L'H}{=} \lim_{u \to 0^+} \frac{sen(u)}{2u} = \frac{1}{2}$.

d) $\lim_{x \to \infty} \frac{3x^4 - x^3 + x^2 + 1}{\frac{3}{2}x^4 - 5x + 1} \overset{L'H}{=} \lim_{x \to \infty} \frac{12x^3 - 3x^2 + 2x}{6x^3 - 5}$

$\overset{L'H}{=} \lim_{x \to \infty} \frac{36x^2 - 6x + 2}{18x^2} \overset{L'H}{=} \lim_{x \to \infty} \frac{72x - 6}{36x}$

$\overset{L'H}{=} \lim_{x \to \infty} \frac{72}{36} = \frac{1}{2}$