P1 $\quad x, y \geq 0 \;/\; \overset{(1)}{x + y = 10} \;/\; S = x^2 + y^2$

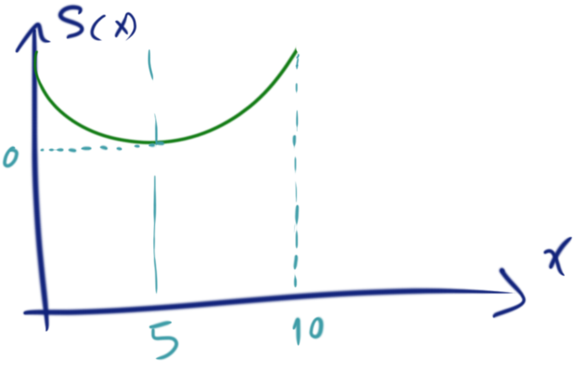
a) de (1), $y = 10 - x$. en (2), $S(x) = x^2 + (10-x)^2$

donde $x \in [0, 10]$.

b) $S(x) = 2x^2 - 20x + 100 = 2(x^2 - 10x + 50)$

$= 2(x^2 - 2 \cdot 5x + 50) = 2((x-5)^2 + 25)$

$= \boxed{2(x-5)^2 + 50}$



c) S mínimo donde

$\boxed{x = y = 5}$, $\boxed{S(5) = 50}$

d) (1) S decreciente en $[0, 5]$
(estrictamente)

dem: sean $x_1, x_2 \in [0, 5]$ tq $x_1 < x_2$,

$x_1 < x_2 \leq 5 \quad / -5$

$x_1 - 5 < x_2 - 5 \leq 0 \quad / \, ()^2$

$(x_1-5)^2 > (x_2-5)^2 \geq 0 \quad / \cdot 2$

$2(x_1-5)^2 > 2(x_2-5)^2 \quad / +50$

$$2(x_1-5)^2+50 > 2(x_2-5)^2+50 \quad /\overset{def}{S(x)}$$

$$S(x_1) > S(x_2) \quad \square$$

(2) S decreciente en $[5,10]$
(estrictamente)

dem: sean $x_1, x_2 \in [5,10]$ t.q $x_1 < x_2$,

$$5 \leq x_1 < x_2 \quad / -5$$

$$0 \leq x_1 - 5 < x_2 - 5 \quad / ()^2$$

$$0 \leq (x_1-5)^2 < (x_2-5)^2 \quad / \cdot 2$$

$$2(x_1-5)^2 < 2(x_2-5)^2 \quad / +50$$

$$2(x_1-5)^2+50 < 2(x_2-5)^2+50 \quad /\overset{def}{S(x)}$$

$$S(x_1) < S(x_2) \quad \square$$

P2 $f: \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$ | $g: \mathbb{R} - \{\frac{2}{3}\} \to \mathbb{R}$, $g(x) = \dfrac{4x-3}{2-3x}$

a)
$$g(x) = \frac{4x-3}{2-3x} = -\left(\frac{4x-3}{3x-2}\right) = -4\left(\frac{x-\frac{3}{4}}{3x-2}\right) = \frac{-4}{3}\left(\frac{3x-\frac{9}{4}}{3x-2}\right)$$

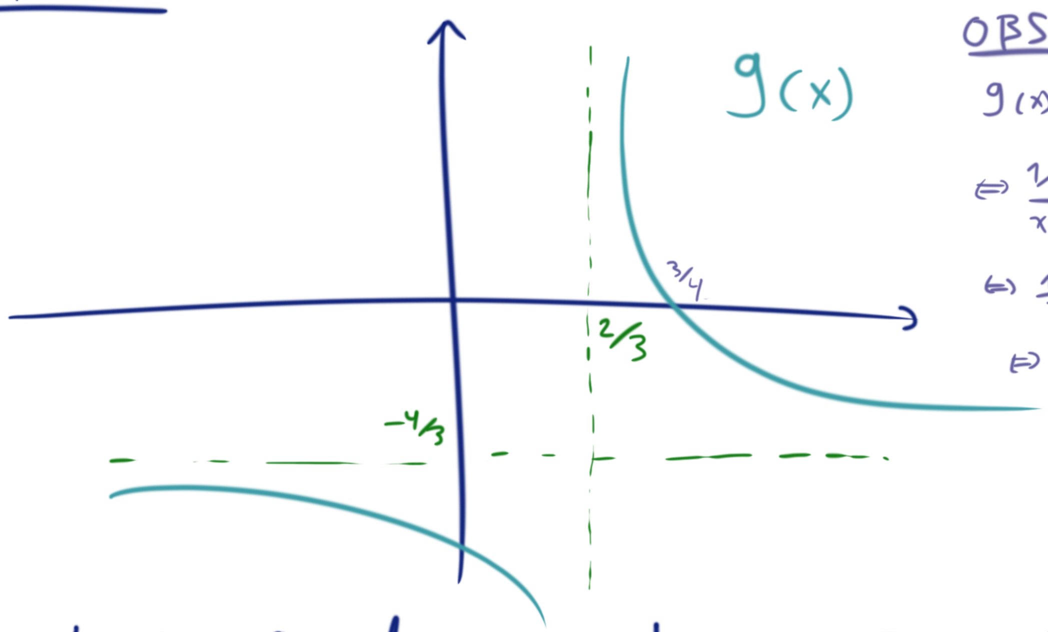
$$= \frac{-4}{3}\left(\frac{3x-2}{3x-2} + \frac{-1/4}{3x-2}\right) = \frac{-4}{3}\left(1 + \frac{-1/4}{3x-2}\right) = \frac{-4}{3} + \frac{1/3}{3x-2} \cdot \frac{1/3}{1/3}$$

$$= \frac{-4}{3} + \frac{1/9}{x-2/3}$$

$a = 1/9 \qquad k = -4/3$

$h = 2/3$

gráfico :



OBS :

$g(x) = 0 \iff \frac{-4}{3} + \frac{1/9}{x - \frac{2}{3}} = 0$

$\iff \frac{1/9}{x - \frac{2}{3}} = \frac{4}{3}$

$\iff \frac{1}{3} = 4x - \frac{8}{3}$

$\iff 4x = 3 \iff \boxed{x = \frac{3}{4}}$

b) hipótesis: g decreciente en $]-\infty, 2/3[\cup]2/3, \infty[$

dem:

(1) $]-\infty, 2/3[$ | sean $x_1, x_2 < \frac{2}{3}$ tq $x_1 < x_2$

Luego:

$$x_1 < x_2 < \frac{2}{3} \quad \Big/ -\frac{2}{3}$$

$$x_1 - \frac{2}{3} < x_2 - \frac{2}{3} < 0 \quad \Big/ ()^{-1}$$

$$\frac{1}{x_1 - \frac{2}{3}} > \frac{1}{x_2 - \frac{2}{3}} \quad \Big/ \cdot \frac{1}{9}$$

$$\frac{1/9}{x_1 - \frac{2}{3}} > \frac{1/9}{x_2 - 2/3} \quad \Big/ -\frac{4}{3}$$

$$\frac{-4}{3} + \frac{1/9}{x_1 - 2/3} > \frac{-4}{3} + \frac{1/9}{x_2 - 2/3} \quad \Big/ \text{def } g(x)$$

$$g(x_1) > g(x_2)$$

$\square$

(2) $]2/3, \infty[$ | sean $x_1, x_2 > \frac{2}{3}$ tq $x_1 < x_2$

luego:

$$\frac{2}{3} < x_1 < x_2 \quad / -\frac{2}{3}$$

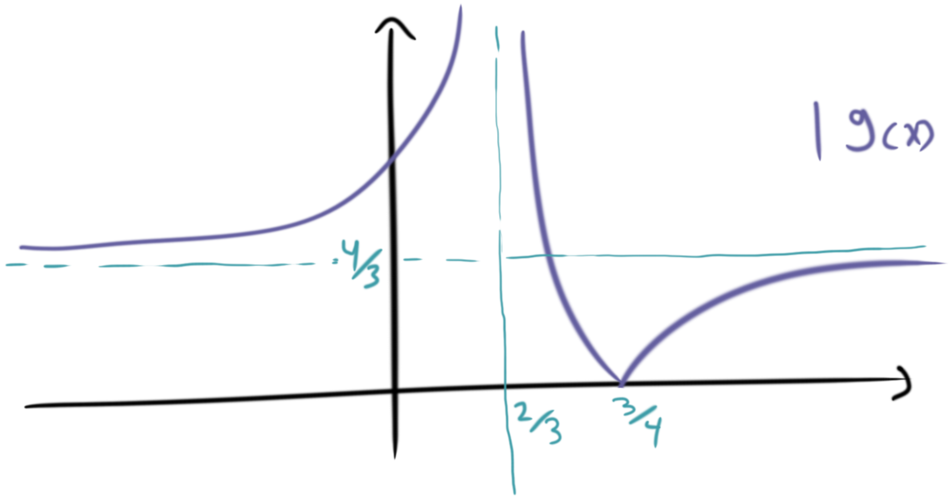
$$0 < x_1 - \frac{2}{3} < x_2 - \frac{2}{3} \quad / ()^{-1}$$

$$\frac{1}{x_1 - \frac{2}{3}} > \frac{1}{x_2 - \frac{2}{3}} \quad / \cdot \frac{1}{9} \quad / -\frac{4}{3}$$

$$\frac{-4}{3} + \frac{1/9}{x_1 - \frac{2}{3}} > \frac{-4}{3} + \frac{1/9}{x_2 - \frac{2}{3}} \quad / \text{def } g(x)$$

$$g(x_1) > g(x_2) \quad \square$$

c) $$(f \circ g)(x) = f(g(x)) = |g(x)| = \left| -\frac{4}{3} + \frac{1/9}{x - \frac{2}{3}} \right|$$



(reflejamos los intervalos negativos de g) ☺

d) $Im(g) = [0, \frac{4}{3}[$ (acotado)

$(x \geq \frac{3}{4})$

P3 | (1) $q(p) = 100.000 - 200p$

(2) $C(q) = 150.000 + 100q + \frac{3}{1000} q^2$

(3) $R = p \cdot q$

(4) $U = R - C$

a) Expresar $R(q)$

$R = q \cdot p$, pero de (1), $p(q) = \frac{100.000 - q}{200}$

luego, $R(q) = q \cdot p(q) = q \cdot \frac{100.000 - q}{200}$ //

b) Expresar $U(q)$

$U = R - C$, pero de (2):

$C(q) = 150.000 + 100q + \frac{3}{1000} q^2$

luego, usando (a),

$$U(q) = R(q) - C(q) = \frac{100.000q - q^2}{200} - C(q)$$

$$= 500q - \frac{1}{200} q^2 - \left(150.000 + 100q + \frac{3}{1000} q^2\right)$$

$$= -\frac{8}{1000} q^2 + 400q - 150.000$$

$$U(q) = -\frac{1}{125} q^2 + 400q - 150.000 \, //$$

$\hookrightarrow (-)$, luego, $\frac{-b}{2a}$ entrega máximo.

c) $$q_{max} = \frac{+\cancel{400}^{200}}{\cancel{2} \cdot \frac{+1}{125}} = 200 \cdot 125 = \underline{25.000}$$

$$P(q_{max}) = \frac{100.000 - 25.000}{200} = \frac{75.0\cancel{00}}{2\cancel{00}} = \underline{375}$$

$\underline{P4}$ $$f(x) = |x+3| + |x-4|$$

a) $A = \{ x \in \mathbb{R} \,/\, f(x) < 18 \}$

<u>condición:</u>

$$|x+3| + |x-4| < 18 \quad / \quad \text{P.C.} \quad x=-3, \; x=4$$

<u>en $]-\infty, -3]$:</u>

$x+3 \leq 0 \longrightarrow |x+3| = -(x+3)$

$x-4 < 0 \longrightarrow |x-4| = -(x-4)$

nos queda; $-x-3+4-x < 18$

$$\Rightarrow -2x < 17 \iff x > \frac{-17}{2}$$

$$\therefore x \in \left]-\frac{17}{2}, \infty\right[\cap \left]-\infty, -3\right] = \left]\frac{-17}{2}, -3\right]$$

en $]-3, 4]$:

$x+3 > 0 \longrightarrow |x+3| = x+3$

$x-4 \leq 0 \longrightarrow |x-4| = -(x-4)$

nos queda;

$$\cancel{x}+3-4-\cancel{x} < 18 \iff -1 < 18$$

$$\therefore x \in \mathbb{R} \cap]-3,4] =]-3,4]$$

en $]4, \infty[$:

$x+3 > 0 \longrightarrow |x+3| = x+3$

$x-4 > 0 \longrightarrow |x-4| = x-4$

nos queda:

$$x+3+x-4 < 18$$

$$2x < 19 \iff x < \frac{19}{2}$$

$$\therefore x \in]-\infty, \tfrac{19}{2}[\cap]4, \infty[$$

$$=]4, \tfrac{19}{2}[$$

luego,

$$A =]\tfrac{-17}{2}, -3] \cup]-3, 4] \cup]-4, \tfrac{19}{2}[$$

$$=]\tfrac{-17}{2}, \tfrac{19}{2}[\,//$$

C.S.: $[\frac{19}{2}, \infty[$

C.I: $]-\infty, \frac{-17}{2}]$

$\sup(A) = \frac{19}{2}$

$\inf(A) = \frac{-17}{2}$

$\max(A) \nexists$

$\min(A) \nexists$