$$P(x) = x^5 + x^4 + x^3 + x^2 + x + 1$$

$$q(x) = x^2 + x + 1$$

emcontrar cuociemte y resto de

$$P(x) : q(x)$$

$$\frac{P(x)}{q(x)} = \frac{x^5 + x^4 + x^3 + x^2 + x + 1}{x^2 + x + 1} = \frac{x^3(x^2 + x + 1) + x^2 + x + 1}{x^2 + x + 1}$$

$$= \frac{\cancel{(x^2 + x + 1)}(x^3 + 1)}{\cancel{(x^2 + x + 1)}} = x^3 + 1$$

$$\Rightarrow P(x) = (x^3 + 1)(x^2 + x + 1)$$

Sabiendo que $c=-4$ es raiz de

$P(x) = x^3+3x^2-10x-24$, factoricelo.

$$x^3+3x^2-10x-24 : (x+4) = x^2-x-6$$

$$\underline{-(x^3+4x^2)}$$

$$-x^2-10x$$

$$\underline{-(-x^2-4x)}$$

$$-6x-24$$

$$\underline{(-6x-24)}$$

$$0 \,//$$

$$\rightarrow x^3+3x^2-10x-24 = (x+4)\underbrace{(x^2-x-6)}_{(x-3)(x+2)}$$

$$P(x) = \underline{(x+4)(x-3)(x+2)}$$

Teoremas:

① Teo. del factor:

$P(x)$ div. por $(x-k) \iff P(k)=0$

② Cr. de raíces racionales

Sea $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ a coeficientes enteros. Si $\frac{c}{d}$ es raíz de $P(x)$, entonces $c | a_0$ y $d | a_n$

Ej: factorizar $P(x) = 2x^3 + 5x^2 - x - 1$

$div(-1) = \{1, -1\}$

$div(2) = \{1, -1, 2, -2\}$

candidatos: $\pm 1, \pm \frac{1}{2}$

$x=1$

$P(1) = 2+5-1-1 = 5 \neq 0$

$x=-1$ $-2+5+1-1 = 3 \neq 0$

$x=\frac{1}{2}$

$2 \cdot \frac{1}{8} + 5 \cdot \frac{1}{4} - \frac{1}{2} - 1 = \frac{1}{4} + \frac{5}{4} - \frac{1}{2} - 1$

$= \frac{6}{4} - \frac{1}{2} - 1 = \frac{3}{2} - \frac{1}{2} - 1 = 1 - 1 = 0$ ✓

∴ $P(x)$ es div. por $(x - \frac{1}{2})$

$$2x^3+5x^2-x-1 : \left(x-\frac{1}{2}\right) = 2x^2+6x+2$$

$$-(2x^3-x^2)$$

$$6x^2-x$$

$$-(6x^2-3x)$$

$$2x-1$$

$$-(2x-1)$$

$$0 \,//$$

$$\overbrace{2x^2+6x+2}^{2(x^2+3x+1)}$$

$$P(x) = \left(x-\frac{1}{2}\right)\left(2x^2+6x+2\right)$$

$$=(2x-1)(x^2+3x+1) \quad (\pm 1)$$

$$\Delta = 9-4 = 5 > 0$$

$$x_{1,2} = \frac{-3\pm\sqrt{5}}{2} = \begin{cases} x_1 = \frac{-3+\sqrt{5}}{2} \\ x_2 = \frac{-3-\sqrt{5}}{2} \end{cases}$$

$$\left(x - \frac{-3+\sqrt{5}}{2}\right)\left(x+\frac{3+\sqrt{5}}{2}\right)$$

$$P(x) = (2x-1)\left(x+\frac{3-\sqrt{5}}{2}\right)\left(x+\frac{3+\sqrt{5}}{2}\right) \checkmark$$

Ejercicio:

determime k para que $x^3-2x^2+kx+18$ sea divisible por $(x-3) \Leftrightarrow P_{(3)}=0$

¿Es cierto que para todo $a \in \mathbb{R}$ existe um k tal que este polimomio es divisible por $(x-a)$?

forma 1: $x^3-2x^2+kx+18 : (x-3) = x^2+x+(k+3)$

$$
\begin{array}{l}
-(x^3-3x^2) \\
\hline
x^2+kx \\
-(x^2-3x) \\
\hline
(k+3)x+18 \\
-((k+3)x-3(k+3)) \\
\hline
18+3(k+3) \overset{!}{=} 0 \quad / \cdot \frac{1}{3}
\end{array}
$$

$$6+k+3 \overset{!}{=} 0$$

$$\boxed{k=-9}$$

forma 2:

$P_{(x)}=x^3-2x^2+kx+18$

impongo que $P_{(3)} \overset{!}{=} 0 \,/\, 27-\cancel{18}+3k+\cancel{18} \overset{!}{=} 0$

$$3k=-27 \Rightarrow \boxed{k=-9}$$