1) a)

$f(x) = x^3+x^2-x+1$, en $x=1$

$$f'(1)=\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^3+x^2-x+1-2}{x-1}$$

$$= \lim_{x\to 1} \frac{x^3+x^2-x-1}{x-1} = \lim_{x\to 1} \frac{x^2(x+1)-(x+1)}{x-1}$$

$$= \lim_{x\to 1} \frac{(x+1)^2\cancel{(x-1)}}{\cancel{(x-1)}} = 4$$

$\therefore f'(1)=4$

b) $$g'(0)=\lim_{x\to 0} \frac{\frac{x^2+x}{x^2+x+1}-0}{x-0} = \lim_{x\to 0} \frac{\overbrace{x^2+x}^{x(x+1)}}{x(x^2+x+1)}$$

$$= \lim_{x\to 0} \frac{x+1}{x^2+x+1} = 1$$, Por lo tanto, $g'(0)=1$.

c) $$h'(a) = \lim_{x\to a} \frac{\sqrt{x^2+5}-\sqrt{a^2+5}}{x-a} = \lim_{x\to a} \frac{(x^2+5)-(a^2+5)}{(x-a)(\sqrt{x^2+5}+\sqrt{a^2+5})}$$

$$= \lim_{x\to a} \frac{(x+a)\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{x^2+5}+\sqrt{a^2+5})} = \frac{2a}{2\sqrt{a^2+5}} = \frac{a}{\sqrt{a^2+5}}$$ //

d)

$$I'(0)= \lim_{h\to 0} \frac{\tan(0+h)-\tan(0)}{h} = \lim_{h\to 0} \frac{\tan(h)}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen}(h)}{h} \cdot \frac{1}{\cos(h)} \overset{A.L.}{=} \lim_{h\to 0} \frac{\operatorname{sen}(h)}{h} \cdot \lim_{h\to 0} \frac{1}{\cos(h)}$$

$$= 1 \cdot \frac{1}{1} = 1$$ //

2)

a) $\lim_{x\to 5} \dfrac{\frac{1}{2x-1}-\frac{1}{9}}{x-5} = f'(5)$, con $f(x)=\dfrac{1}{2x-1}$

$f'(x)=\dfrac{-2}{(2x-1)^2}$, luego $f'(5)=\dfrac{-2}{81}$

$\therefore \lim_{x\to 5} \dfrac{\frac{1}{2x-1}-\frac{1}{9}}{x-5} = \dfrac{-2}{81}$

b) $\lim_{h\to 0} \dfrac{\sqrt[3]{27+h}-3}{h} = f'(27)$, con $f(x)=\sqrt[3]{x}$

$f'(x)=\frac{1}{3}x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$, $f'(27)=\dfrac{1}{3\cdot 3^2}=\dfrac{1}{27}$

$\therefore \lim_{x\to 0} \dfrac{\sqrt[3]{27+h}-3}{h} = \dfrac{1}{27}$

c) $\lim_{x\to -1} \dfrac{(x^2+1)^2(x^2-1)^2-16}{x+1} = f'(-1)$, con

$f(x)=(x^2+1)^2(x^2-1)^2 = (x^4-1)^2 = x^8-2x^4+1$

$f'(x)=8x^7-8x^3$, $f'(-1)=-8+8=0$

3 a) $f(x)=(3x^2+1)^3 = (27x^6+3(3x^2)^2+3(3x^2)+1)$

$= 27x^6+27x^4+9x^2+1$

$f'(x)=162x^5+108x^3+18x$

b) $g(x)=\dfrac{2x^2-3x+1}{x^2+x-2}$

$g'(x)=\dfrac{(4x-3)(x^2+x-2)-(2x^2-3x+1)(2x+1)}{(x^2+x-2)^2}$

$$g'(x) = \frac{4x^3+4x^2-8x-3x^2-3x+6-[4x^3+2x^2-6x^2-3x+2x+1]}{(x^2+x-2)^2}$$

$$g'(x) = \frac{5x^2-10x+5}{(x^2+x-2)^2} = \frac{5(x^2-2x+1)}{(x^2+x-2)^2} = \frac{5\cancel{(x-1)^2}}{(x+2)^2\cancel{(x-1)^2}}$$

$$= \frac{5}{(x+2)^2}$$

$x \neq 1$

Forma 2: $g(x) = \frac{(2x-1)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \frac{2x-1}{x+2}$

$$g'(x) = \frac{2(x+2)-(2x-1)\cdot 1}{(x+2)^2} = \frac{5}{(x+2)^2} //$$

c) $h(x) = \tan(x)\cdot(x^2+x+1)$

$h'(x) = \sec^2(x)(x^2+x+1) + \tan(x)\cdot(2x+1) //$

d) $I(x) = \frac{(x^2+1)\sqrt{x}}{(x^2-1)^3 \operatorname{sen}(x)}$

$$= \frac{\left(2x\sqrt{x}+(x^2+1)\frac{1}{2\sqrt{x}}\right)\cdot(x^2-1)^3\operatorname{sen}(x) - (x^2+1)\sqrt{x}\cdot\left[3(x^2-1)\,2x\operatorname{sen}(x) + (x^2-1)^3\cdot\cos(x)\right]}{(x^2-1)^6\operatorname{sen}^2(x)}$$

$//$