Taller 12

1 a) PDI $\sec^4(\alpha) - \sec^2(\alpha) = \tan^2(\alpha)\sec^2(\alpha)$

$$\sec^4(\alpha) - \sec^2(\alpha) = \sec^2(\alpha)\left(\sec^2(\alpha) - 1\right)$$

$$= \sec^2(\alpha)\left(\frac{1}{\cos^2\alpha} - 1\right)$$

$$= \sec^2(\alpha)\left(\frac{1-\cos^2\alpha}{\cos^2\alpha}\right) \quad / \ \operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$= \sec^2(\alpha)\left(\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}\right)$$

$$= \sec^2(\alpha)\cdot\tan^2(\alpha).$$

Para que no se anulen estas expresiones, $\cos(\alpha) \neq 0 \Leftrightarrow \alpha \neq \frac{\pi}{2} + k\pi;\ k \in \mathbb{Z}$

b) Si α es agudo, entonces $0 < \alpha < \frac{\pi}{2}$ y podemos usar a).

Luego

$$\cos^4(\alpha)\left[\underbrace{\sec^4(\alpha) - \sec^2(\alpha)} - \tan^4(\alpha) - \tan^2(\alpha)\right] = 1 - 2\cos\alpha$$

$$\Rightarrow \cos^4(\alpha)\left[\tan^2(\alpha)\sec^2(\alpha) - \tan^2(\alpha) - \tan^4(\alpha)\right] = 1 - 2\cos\alpha$$

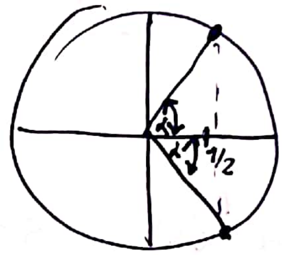
$$\Rightarrow \cos^4(\alpha)\left[\tan^2(\alpha)\left(\sec^2(\alpha) - 1\right) - \tan^4(\alpha)\right] = 1 - 2\cos\alpha$$

/ de a) también se deduce que $\sec^2\alpha - 1 = \tan^2\alpha$

$$\Rightarrow \cos^4(\alpha)\left[\underbrace{\tan^4(\alpha) - \tan^4(\alpha)}_{0}\right] = 1 - 2\cos\alpha$$

$\Rightarrow \quad 1-2\cos\alpha = 0$

$\Rightarrow \quad \cos\alpha = \dfrac{1}{2}$



$$\Rightarrow \quad \alpha = \begin{cases} \pi/3 + 2\pi k \\ 2\pi - \pi/3 + 2\pi k \end{cases}, \; k \in \mathbb{Z}$$

Como $0 < \alpha < \frac{\pi}{2}$, $\alpha = \frac{\pi}{3}$ y así

$$\tan^2(\alpha) = \tan^2\left(\tfrac{\pi}{3}\right) = \left(\sqrt{3}\right)^2 = 3$$

c)

PD| $\operatorname{sen}^4\alpha + \cos^4\alpha = 1 - 2\cos^2(\alpha)\operatorname{sen}^2\alpha$

☆ Tip: Uno empieza del lado con más términos, (usualmente) porque es más fácil reducir términos que hacerlos aparecer.

Luego

$$\begin{aligned} 1 - 2\cos^2\alpha\operatorname{sen}^2\alpha &= \operatorname{sen}^2\alpha + \cos^2\alpha - 2\cos^2\alpha\operatorname{sen}^2\alpha \\ &= \operatorname{sen}^2\alpha + \cos^2\alpha\left(1 - 2\operatorname{sen}^2\alpha\right) \\ &= \operatorname{sen}^2\alpha + (1-\operatorname{sen}^2\alpha)(1-2\operatorname{sen}^2\alpha) \\ &= \operatorname{sen}^2\alpha + 1 - 2\operatorname{sen}^2\alpha - \operatorname{sen}^2\alpha + 2\operatorname{sen}^4\alpha \end{aligned}$$

/ Pues $1 = \operatorname{sen}^2\alpha + \cos^2\alpha$

/ $\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$

Luego

$$1-2\cos^2\alpha\,\text{sen}^2\alpha = \text{sen}^2\alpha + 1 - 2\text{sen}^2\alpha - \text{sen}^2\alpha + \text{sen}^4\alpha + \text{sen}^4\alpha \checkmark$$

$$= \underbrace{1 - 2\,\text{sen}^2\alpha + \text{sen}^4\alpha} + \text{sen}^4\alpha$$

$$= (1-\text{sen}^2\alpha)^2 + \text{sen}^4(\alpha) \quad / \begin{matrix} 1-\text{sen}^2\alpha \\ =\cos^2\alpha \end{matrix}$$

$$= (\cos^2\alpha)^2 + \text{sen}^4\alpha$$

$$= \cos^4\alpha + \text{sen}^4\alpha \quad \square$$

d) PDI $\text{sen}^6(\alpha) + \cos^6(\alpha) = 1 - 3\,\text{sen}^2\alpha\,\cos^2\alpha$

$$1-3\,\text{sen}^2\alpha\cos^2\alpha = \underbrace{1-2\,\text{sen}^2\alpha\cos^2\alpha} - \text{sen}^2\alpha\cos^2\alpha \quad / \text{Por c)}$$

$$= \cos^4\alpha + \text{sen}^4\alpha - \text{sen}^2\alpha\cos^2\alpha$$

$$= 1\cdot(\text{sen}^4\alpha - \text{sen}^2\alpha\cos^2\alpha + \cos^4\alpha)$$

$$= (\text{sen}^2\alpha + \cos^2\alpha)(\text{sen}^4\alpha - \text{sen}^2\alpha\cos^2\alpha + \cos^4\alpha) \quad / \begin{matrix} x^3+y^3 \\ =(x+y) \\ (x^2-xy+y^2)\end{matrix}$$

$$= \text{sen}^6\alpha + \cos^6\alpha \quad \square$$

e) $(\text{sen}^2\alpha - \cos^2\alpha)^2 + (\tan\alpha + \cot\alpha)^{-2} = (\text{sen}^2\alpha - \cos^2\alpha)^2 + \left(\frac{\text{sen}\,\alpha}{\cos\alpha} + \frac{\cos\alpha}{\text{sen}\,\alpha}\right)^{-2}$

$$= (\text{sen}^2\alpha - \cos^2\alpha)^2 + \left(\frac{\text{sen}^2\alpha + \cos^2\alpha}{\cos\alpha\cdot\text{sen}\,\alpha}\right)^{-2}$$

$$= (\text{sen}^2\alpha - \cos^2\alpha)^2 + \cos^2\alpha\,\text{sen}^2\alpha$$

$$= \text{sen}^4\alpha - 2\cos^2\alpha\,\text{sen}^2\alpha + \cos^4\alpha + \cos^2\alpha\,\text{sen}^2\alpha$$

$$= \text{sen}^4\alpha - \cos^2\alpha\,\text{sen}^2\alpha + \cos^4\alpha$$

Luego,

$$\text{sen}^4\alpha - \text{sen}^2\alpha\cos^2\alpha + \cos^4\alpha = \underbrace{(\text{sen}^2\alpha + \cos^2\alpha)}^{1}\cdot(\text{sen}^4\alpha - \text{sen}^2\alpha\cos^2\alpha + \cos^2$$

$$= \text{sen}^6\alpha + \cos^6\alpha$$

Por d)

2) a) $\cos(2x) = 1 + 4\,\text{sen}\,x$ / Como $\cos(2x) = \ {}^2x \quad {}^2x$

$= \ {}^2x \quad (1 - \text{sen}^2 x)$

$$1 - 2\,\text{sen}^2(x) = 1 + 4\,\text{sen}\,x$$

$$2\,\text{sen}^2 x + 4\,\text{sen}\,x = 0$$

$$2\,\text{sen}\,x\,(\text{sen}\,x + 2) = 0$$

$2\,\text{sen}\,x = 0 \quad \vee \quad \text{sen}\,x + 2 = 0$

$\Rightarrow \text{sen}\,x = 0 \quad \vee \quad \text{sen}\,x = -2$

$\Rightarrow x = \begin{cases} 0 + 2\pi k \\ \pi + \pi k \end{cases} \quad \vee \quad$ Como $-1 \leq \text{sen}\,x \leq 1$, no hay soluciones en este caso

$\Rightarrow x = \{\pi k, \ k \in \mathbb{Z}.$

Como buscamos $x \in [0, 2\pi]$

$$x = 0, \ x = \pi \ \text{ó} \ x = 2\pi$$

b) $\cos(x) + \cos(3x) = 0$

Primero, veamos como escribir $\cos(3x)$ en términos de $\cos x$.

$$\cos(3x) = \cos(2x + x)$$

$$= \underbrace{\cos(2x)}\cos(x) - \underbrace{\operatorname{sen}(2x)}\operatorname{sen}x$$

$$= \left[\cos^2(x) - \operatorname{sen}^2 x\right]\cos(x) - \left[2\operatorname{sen}x\cos x\right]\operatorname{sen}x$$

$$= \left[\cos^2 x - (1-\cos^2 x)\right]\cos(x) - 2\cos x \operatorname{sen}^2 x$$

$$= \left[2\cos^2(x) - 1\right]\cos(x) - 2\cos(x)(1-\cos^2 x)$$

$$= 2\cos^3(x) - \cos(x) - 2\cos(x) + 2\cos^3 x$$

$$= 4\cos^3(x) - 3\cos(x)$$

Luego la ecuación queda

$$\cos(x) + 4\cos^3(x) - 3\cos(x) = 0$$

$$\Rightarrow 4\cos^3(x) - 2\cos(x) = 0$$

$$\Rightarrow 2\cos(x)\left[2\cos^2 x - 1\right] = 0$$

$$\Rightarrow \cos(x)\left[2\cos^2 x - 1\right] = 0$$

$$\Rightarrow \cos x = 0 \quad \vee \quad 2\cos^2 x - 1 = 0$$

$$\Rightarrow \cos x = \frac{\sqrt{2}}{2} \quad \vee \quad \cos x = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow x = \begin{cases} \frac{\pi}{2} + 2\pi k \\ \frac{3\pi}{2} + 2\pi k \end{cases}, k \in \mathbb{Z} \quad \Bigg| \quad x = \begin{cases} \pi/4 + 2\pi k \\ 2\pi - \frac{\pi}{4} + 2\pi k \end{cases} \quad \Bigg| \quad x = \begin{cases} \frac{3\pi}{4} + 2\pi k \\ \frac{5\pi}{4} + 2\pi k \end{cases}$$

~~kk~~

$$\Rightarrow x = \{\pi/2 + \pi k$$

Restringiendo los valores obtenidos a $[0, 2\pi]$, los valores posibles son

$$x \in \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \right\}$$