Taller 13

P1] $xy\dfrac{dy}{dx} - \ln x = 0$

$$y\frac{dy}{dx} = \frac{\ln x}{x}$$

$$\int y\,dy = \int \frac{\ln x}{x}\,dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{1}{2}\ln^2 x + C$$

$$y^2 = \ln^2 x + C$$

$$y(x) = \sqrt{\ln^2 x + C} \qquad y(x) = -\sqrt{\ln^2 x + C}$$

$\int \frac{\ln x}{x}\,dx$

$= \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{1}{2}\ln^2 x + C$

Notar que $(\ln x)' = \frac{1}{x}$

$u = \ln x$

$du = \frac{1}{x}dx$

P2] a). $\sqrt{1-x^2}\,\dfrac{dy}{dx} - \sqrt{1-y^2} = 0$, $y(0) = 1$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \sqrt{1-x^2}$$

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int \frac{1}{\sqrt{1-x^2}}\,dx$$

Como $\text{arcsen}'x = \frac{1}{\sqrt{1-x^2}}$, tenemos

$\text{arcsen}\, y = \text{arcsen}\, x + C$

Como $y(0) = 1$, si $x=0 \Rightarrow y=1$

$\text{arcsen}\, 1 = \text{arcsen}\, 0 + C$

$\frac{\pi}{2} = 0 + C$

Así, $C = \frac{\pi}{2}$ y

$$\arcsin y = \arcsin x + \frac{\pi}{2} \quad / \operatorname{sen}(\ )$$

$$y = \operatorname{sen}\left(\arcsin x + \frac{\pi}{2}\right) \quad / \ \operatorname{sen}\alpha+\beta = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$$

$$y = x \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + \cos(\arcsin x) \underbrace{\operatorname{sen}\left(\frac{\pi}{2}\right)}_{1}$$

$$y = \cos(\operatorname{arsen} x) \quad / \ \text{como } \cos(x) = \sqrt{1-\operatorname{sen}^2 x}$$

$$y_{(x)} = \sqrt{1 - x^2}$$

Una forma de verificar el resultado, es comprobar si satisface la condición inicial de $y(0) = 1$

$$1 \stackrel{?}{=} \sqrt{1-0^2} \quad \checkmark$$

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P2) b)

$$x e^{-y} \operatorname{sen} x \, dx - 2dy = 0$$

$$x \operatorname{sen} x \, dx \cdot e^{-y} = 2dy \quad / \cdot e^{y}$$

$$x \operatorname{sen} x \, dx = 2e^{y} dy$$

$$\underbrace{\int x \operatorname{sen} x \, dx}_{\text{Por partes}} = \underbrace{\int 2e^{y} dy}_{\text{directa}}$$

$\int x \operatorname{sen} x \, dx$ → ILATE, $u = x$, $dv = \operatorname{sen} x \, dx$; $du = dx$, $v = -\cos x$

$$\int x \operatorname{sen} x \, dx = -x\cos x - \int -\cos x \, dx$$

$$= -x\cos x + \operatorname{sen} x + C$$

$$\Rightarrow -x\cos x + \operatorname{sen} x + C = 2e^{y}$$

$$\frac{-x\cos x + \operatorname{sen} x + C}{2} = e^{y}$$

$$\ln\left|\frac{\operatorname{sen} x - x\cos x + C}{2}\right| = y(x)$$

$\underline{y\left(\frac{\pi}{2}\right) = 1}$

$$\ln\left|\frac{\operatorname{sen}\pi/2}{2} - \frac{\frac{\pi}{2}\cos\frac{\pi}{2}}{2} + \frac{C}{2}\right| = 1$$

$$\ln\left|\frac{1}{2} + \frac{C}{2}\right| = 1$$

$$\frac{1}{2} + \frac{C}{2} = e$$

$$1 + C = 2e$$

$$C = 2e - 1$$

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