

$$\begin{array}{l} -2x_1 + x_2 + x_3 - 2x_4 + x_5 = 6 \\ x_1 + x_2 + x_4 - 2x_5 = -1 \\ 3x_1 - x_2 + x_3 + 4x_4 - 6x_5 = 0 \end{array}$$

## Control 2 de Algebra y Geometría

9 de Abril de 2015

Nombre:.....

1. Demuestre:

a) Sean  $A$  y  $B$  matrices de orden  $n$  invertibles tales que  $A+B$  es invertible.  
Pruebe que  $A^{-1}+B^{-1}$  es invertible y  $(A^{-1}+B^{-1})^{-1} = A(A+B)^{-1}B$ .

b) Sean  $M$  y  $N$  matrices invertibles y  $M$  una matriz simétrica. Demuestre que si

$$M^t N^{-1} + (N^{-1} M^t)^t - (N^t M^{-1})^{-1} = I_n \text{ entonces } M = N.$$

$$a) A(A+B)^{-1}B = A[B^{-1}(A+B)]^{-1}$$

$$= [B^{-1}(A+B) \cdot A^{-1}]^{-1}$$

$$= [B^{-1}A^{-1} + B^{-1}B^{-1}]^{-1}$$

$$= (B^{-1}I + I A^{-1})^{-1}$$

$$= (A^{-1} + B^{-1})^{-1}$$

Como la inversa es única, por lo que:

$$(A^{-1} + B^{-1}) = A(A+B)^{-1}B$$

$$b) M^t N^{-1} + (N^{-1} M^t)^t - (N^t M^{-1})^{-1} = I_n$$

$$N^t N^{-1} + N(N^{-1})^t - M \cdot (N^t)^{-1} = I_m$$

$$N^t N^{-1} + N(N^t)^{-1} - M(N^t)^{-1} = I_m$$

$$N^t N^{-1} = I_m \quad | \cdot N$$

$$N^t = N \quad / N = M^t$$

$$\underline{M = N}$$

2. Considere el siguiente sistema lineal:

$$\begin{aligned}-2x_1 + x_2 + x_3 - 2x_4 + x_5 &= 6 \\ x_1 + x_2 + x_4 - 2x_5 &= -1 \\ 3x_1 - x_2 + x_3 + 4x_4 - 6x_5 &= 0\end{aligned}$$

0,5

a) Escriba el sistema de forma matricial  $Ax = b$ .

b) Resuelva el sistema lineal escalonando la matriz aumentada  $(A|b)$ , dando la solución de manera matricial (si existe).

a)

$$\left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 1 & 1 & 0 & 1 & -2 & -1 \\ 3 & -1 & 1 & 4 & -6 & 0 \end{array} \right) \xrightarrow{F_1 \leftrightarrow F_2} \left( \begin{array}{cccc|c} 1 & 1 & 0 & 1 & -2 & 1 & -1 \\ -2 & 1 & 1 & -2 & 1 & 1 & 6 \\ 3 & -1 & 1 & 4 & -6 & 0 \end{array} \right)$$

$$\xrightarrow{\quad} \left( \begin{array}{cccc|c} 1 & 1 & 0 & 1 & -2 & 1 & -1 \\ 0 & 3 & 1 & 0 & -3 & 1 & 4 \\ 0 & -4 & 1 & 1 & 0 & 1 & 3 \end{array} \right) \xrightarrow{\quad} \left( \begin{array}{cccc|c} 1 & 1 & 0 & 1 & -2 & 1 & -1 \\ 0 & 3 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & \frac{1}{3} & 1 & -4 & 1 & \frac{25}{3} \end{array} \right)$$

$$\xrightarrow{\quad} \left( \begin{array}{cccc|c} 1 & 1 & 0 & 1 & -2 & 1 & -1 \\ 0 & 3 & 1 & 0 & -3 & 1 & 4 \\ 0 & 0 & \frac{1}{3} & 1 & -4 & 1 & \frac{25}{3} \end{array} \right) \quad \checkmark \quad \begin{matrix} 1 \\ x_4 \text{ y } x_5 \text{ libres} \end{matrix}$$

$$x_3 = \frac{25}{4} - \frac{3x_4}{4} + \frac{12x_5}{4}$$

$$3x_2 = 4 - x_3 + 3x_5$$

$$3x_2 = 4 - \frac{25}{4} + \frac{3}{4}x_4 - \frac{12}{4}x_5 + 3x_5$$

$$3x_2 = \frac{3}{4} + \frac{3}{4}x_4 + \frac{9}{4}x_5 \quad 2$$

$$x_2 = \frac{1}{4} + \frac{1}{4}x_4 + \frac{3}{4}x_5$$

1 pts

$$x_1 = -1 - x_2 - x_4 + 2x_5$$

$$x_1 = -1 - \cancel{\frac{1}{4}} - \frac{1}{4}x_4 - \frac{3}{4}x_5 - \cancel{x_4} + \frac{2}{4}x_5$$

$$x_1 = -\frac{8}{4} - \frac{8}{4}x_4 + \frac{11}{4}x_5$$

$$X = \begin{pmatrix} -8/7 \\ 1/7 \\ 25/7 \\ 0 \\ 0 \end{pmatrix} + X_4 \begin{pmatrix} 8/7 \\ 1/7 \\ -3/7 \\ 1 \\ 0 \end{pmatrix} + X_5 \begin{pmatrix} 11/7 \\ 3/7 \\ 42/7 \\ 0 \\ 1 \end{pmatrix}$$

0,5

Ora se poate

$$1 = 3x_2 - x_3 + 5x_4 + 7x_5$$

$$0 = 2x_2 - x_3 + 5x_4 + 7x_5$$

$$d = x_5$$

Întrucât obiectivul este să adunăm la  
tută liniște să rezolvam sistemul obținut de la înmulțirea cu  
(-1) a sistemului original și obținem

$$\left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 1 & 1 & x_5 \\ 0 & 0 & x_2 & 1 & 1 & x_5 \end{array} \right) \xrightarrow{\text{adunare}} \left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 1 & 0 & x_5 \\ 0 & 0 & x_2 & 1 & 1 & x_5 \end{array} \right)$$

$$\left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 1 & 0 & x_5 \\ 0 & 0 & x_2 & 1 & 1 & x_5 \end{array} \right) \xrightarrow{\text{adunare}} \left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 0 & 1 & 0 \\ 0 & 0 & x_2 & 1 & 1 & x_5 \end{array} \right)$$

$$\left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 0 & 1 & 0 \\ 0 & 0 & x_2 & 1 & 1 & x_5 \end{array} \right) \xrightarrow{\text{adunare}} \left( \begin{array}{ccccc} 1 & 1 & x_2 & 0 & 1 & 1 \\ 0 & 1 & x_2 & 0 & 1 & 0 \\ 0 & 0 & x_2 & 1 & 0 & x_5 \end{array} \right)$$

adunare

$$\boxed{e^X \frac{d}{dt} + e^X \frac{d}{dt} - \frac{d}{dt} e^X = e^X}$$

$$e^X \frac{d}{dt} + e^X \frac{d}{dt} - e^X \frac{d}{dt} = e^X$$

$$e^X \frac{d}{dt} + e^X \frac{d}{dt} - e^X \frac{d}{dt} = e^X$$

$$\boxed{e^X \frac{d}{dt} + e^X \frac{d}{dt} + \frac{d}{dt} = e^X}$$

$$e^X \frac{d}{dt} + e^X \frac{d}{dt} - e^X \frac{d}{dt} = e^X$$

$$e^X \frac{d}{dt} + e^X \frac{d}{dt} - e^X \frac{d}{dt} = e^X$$

$$1 + 1 + 1 - 1 = 2$$

$$\begin{array}{c}
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 1 & 1 & 0 & 1 & -2 & 1 \\ 3 & -1 & 1 & 4 & -6 & 0 \end{array} \right) \rightarrow \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 1 & 1 & 0 & 1 & -2 & 1 \\ 0 & 4 & 1 & 2 & 6 & 0 \end{array} \right) \\
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3/2 & 1/2 & 0 & -3/2 & 1 \\ 0 & -4 & 1 & 1 & 0 & 3 \end{array} \right) \rightarrow \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3/2 & 1/2 & 0 & -3/2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{array} \right) \\
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3 & 1 & 0 & -3 & 1 \\ 0 & 0 & -1/3 & 1 & -4 & 1/3 \end{array} \right) \rightarrow \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & 3 & 1/2 & 1/3 \end{array} \right)
 \end{array}$$

$$\begin{array}{c}
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 1 & 1 & 0 & 1 & -2 & 1 \\ 3 & -1 & 1 & 4 & -6 & 0 \end{array} \right) \rightarrow \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 1 & 1 & 0 & 1 & -2 & 1 \\ 0 & -4 & 1 & 1 & 0 & 3 \end{array} \right) \\
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3/2 & 1/2 & 0 & -3/2 & 1 \\ 0 & -4 & 1 & 1 & 0 & 3 \end{array} \right) \rightarrow \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3 & 1 & 0 & -3 & 1 \\ 0 & -4 & 1 & 1 & 0 & 3 \end{array} \right) \\
 \left( \begin{array}{cccc|c} -2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & -4 & 1 & 25/3 \end{array} \right)
 \end{array}$$

$$\Rightarrow \left( \begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 & 6 \\ 0 & 3 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & -3 & 1 & 4 \end{array} \right)$$

$$x_3 = \frac{25}{4} - \frac{3x_1}{4} + \frac{12x_2}{4}$$

$$\begin{aligned}
 b) \quad M^t N^{-1} + I_m &= M \cdot (N^{-1})^t \\
 M^t N^{-1} + M(N^{-1})^t - M \cdot (N^{-1})^t &= I_m \\
 M^t N^{-1} + \cancel{M(N^t)^{-1}} - \cancel{M(N^t)^{-1}} &= I_m
 \end{aligned}$$

$$\begin{aligned}
 M^t N^{-1} &= I_m & \text{A. } N \\
 M^t &= N & / M = M^t \\
 M &= N^{-1}
 \end{aligned}$$

$$x_2 = \frac{1}{7} + \frac{1}{4}x_4 + \frac{3}{7}x_5$$

$$-2x_1 + x_2 + x_3 - 2x_4 + x_5 = 6$$

$$-2x_1 + \cancel{\frac{1}{7}x_2} + \cancel{\frac{3}{7}x_5} + 2x_4 \Rightarrow \frac{3}{7}x_1 + \frac{12}{7}x_5 - 2x_4 + x_5 = 6$$

$$-2x_1 + \frac{26}{7} - \frac{2}{7}x_4 \Rightarrow 2x_1 + \frac{15}{7}x_5 + x_5 = 6$$

$$-\frac{42}{7} + \frac{26}{7} - \frac{16}{7}x_4 + \frac{22}{7}x_5 = 2x_1$$

$$-\frac{16}{7}x_4 + \frac{22}{7}x_5 = 2x_1$$

$$\therefore x_1 = \frac{-8}{7} - \frac{8}{7}x_4 + \frac{11}{7}x_5$$