# Pauta Control 2 de Matemáticas 1

Programa de Bachillerato. Universidad de Chile.

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**Tiempo :** 15 **minutos .**

**Nombre:**

**Elija sólo un problema.**

1. Calcule la siguiente suma.

$$\sum_{k=1}^{n}\sum_{i=1}^{7}(2i^2k-20)$$

Solución:

Notar que

$$\begin{aligned}\sum_{k=1}^{n}\sum_{i=1}^{7}(2i^2k-20) &= \sum_{k=1}^{n}\left(\sum_{i=1}^{7}2i^2k-\sum_{i=1}^{7}20\right)\\ &= \sum_{k=1}^{n}\left(2k\sum_{i=1}^{7}i^2-20\sum_{i=1}^{7}1\right)\end{aligned}$$

Recordemos que

$$\sum_{i=1}^{n}i^2=\frac{n(n+1)(2n+1)}{6}$$

entonces se tiene que

$$\begin{aligned}\sum_{k=1}^{n}\left(2k\sum_{i=1}^{7}i^2-20\sum_{i=1}^{7}1\right) &= \sum_{k=1}^{n}\left(2k\frac{7\cdot(7+1)\cdot(2\cdot 7+1)}{6}-20\cdot 7\right)\\ &= \sum_{k=1}^{n}(280k-140)\\ &= 280\sum_{k=1}^{n}k-140\sum_{k=1}^{n}1\end{aligned}$$

Recordemos que

$$\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$$

entonces

$$\begin{aligned} 280\sum_{k=1}^{n} k - 140\sum_{k=1}^{n} 1 &= 280\frac{n(n+1)}{2} - 140n \\ &= 140n(n+1) - 140n \\ &= 140n[(n+1)-1] \\ &= 140n^2 \end{aligned}$$

Por tanto

$$\sum_{k=1}^{n}\sum_{i=1}^{7}(2i^2k - 20) = 140n^2$$

2. Calcule la siguiente suma.

$$\sum_{k=10}^{100}\frac{2}{k^2+4k+3}$$

<u>Solución:</u>

Notar que

$$\sum_{k=1}^{100}\frac{2}{k^2+4k+3} = \sum_{k=1}^{9}\frac{2}{k^2+4k+3} + \sum_{k=10}^{100}\frac{2}{k^2+4k+3}$$

entonces

$$\sum_{k=10}^{100}\frac{2}{k^2+4k+3} = \sum_{k=1}^{100}\frac{2}{k^2+4k+3} - \sum_{k=1}^{9}\frac{2}{k^2+4k+3}$$

Calculemos entonces para cualquier $n \in \mathbb{N}$

$$\sum_{k=1}^{n}\frac{2}{k^2+4k+3}$$

En efecto, observar que

$$\frac{2}{k^2+4k+3} = \frac{1}{k+1} - \frac{1}{k+3}$$

entonces

$$\begin{aligned} \sum_{k=1}^{n}\frac{2}{k^2+4k+3} &= \sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k+3}\right) \\ &= \sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k+2}\right) + \left(\frac{1}{k+2} - \frac{1}{k+3}\right) \\ &= \sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k+2}\right) + \sum_{k=1}^{n}\left(\frac{1}{k+2} - \frac{1}{k+3}\right) \\ &= \left(\frac{1}{1+1} - \frac{1}{n+2}\right) + \left(\frac{1}{1+2} - \frac{1}{n+3}\right) \end{aligned}$$

Por tanto

$$\sum_{k=10}^{100} \frac{2}{k^2+4k+3} = \frac{5}{6} - \left(\frac{1}{102} + \frac{1}{103}\right) - \left(\frac{5}{6} - \left(\frac{1}{11} + \frac{1}{12}\right)\right)$$