Guía 4 Matemática II Resuelta

Programa Académico de Bachillerato

1. Calcule los siguientes límites:

a)

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

Primero,

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x - 0}{1} = \frac{1}{1}$$

b)

$$\lim_{x \to 0} \frac{ln(1 + x)}{x}$$

Primero,

$$\lim_{x \to 0} \frac{ln(1 + x)}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x \to 0} \frac{\frac{1}{1 + x}}{1} = 1$$

c)

$$\lim_{x \to 0} \frac{a^x - 1}{x}$$

Primero,

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x\to 0}\frac{a^x-1}{x}=\lim_{x\to 0}\frac{a^x\ln(a)-0}{1}=\frac{1\ln(a)}{1}=\ln(a)$$

d)

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

Primero,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\lim_{x\to 0}\frac{a^x\ln(a)-b^x\ln(b)}{1}=\ln(a)-\ln(b)$$

e)

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}$$

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}=\lim_{x\to 0}\frac{\frac{ln(1+x)}{\ln(a)}}{x}$$

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}=\frac{1}{\ln(a)}\lim_{x\to 0}\frac{ln(1+x)}{x}$$

Primero,

$$\frac{1}{\ln(a)}\lim_{x\to 0}\frac{ln(1+x)}{x}=\frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\frac{1}{\ln(a)}\lim_{x\to 0}\frac{\frac{1}{1+x}}{1}=\frac{1}{\ln(a)}$$

2. Calcule las siguientes primitivas:

a)

$$\int \ln(x)\, dx$$

$$f(x) = \ln(x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

$$g'(x) = 1 \quad \Rightarrow \quad g(x) = x$$

$$\int \ln(x) \cdot 1dx = xln(x) - \int \frac{1}{x} xdx$$

$$\int \ln(x)\, dx = xln(x) - x + c$$

b)

$$\int \text{Arctg}(x)\, dx$$

$$f(x) = \text{Arctg}(x) \quad \Rightarrow \quad f'(x) = \frac{1}{x^2+1}$$

$$g'(x) = 1 \quad \Rightarrow \quad g(x) = x$$

$$\int \text{Arctg}(x) \cdot 1dx = x\,\text{Arctg}(x) - \int \frac{1}{x^2+1} xdx$$

$$\int \text{Arctg}(x) \cdot 1dx = x\,\text{Arctg}(x) - \int \frac{x}{x^2+1} dx$$

$$\int \text{Arctg}(x)\, dx = x\,\text{Arctg}(x) - \frac{1}{2}\int \frac{2x}{x^2+1} dx$$

$$\int \text{Arctg}(x)\, dx = x\,\text{Arctg}(x) - \frac{1}{2} ln|x^2+1| + c$$

c)

$$\int \sec(x)\,dx$$

Multipliquemos por 1,

$$\int \sec(x)\frac{(\sec(x)+\tan(x))}{(\sec(x)+\tan(x))}dx$$

$$\int \frac{(\sec^2(x)+\sec(x)\tan(x))}{(\sec(x)+\tan(x))}dx = \int \frac{(\sec(x)+\tan(x))'}{(\sec(x)+\tan(x))}dx$$

$$\int \frac{(\sec(x)+\tan(x))'}{(\sec(x)+\tan(x))}dx = \ln|\sec(x)+\tan(x)| + \mathrm{c}$$

d)

$$\int \sec^3(x)\,dx = \int \sec^2(x)\sec(x)dx$$

$$f(x) = \sec(x) \quad \Rightarrow \quad f'(x) = \sec(x)\tan(x)$$

$$g'(x) = \sec^2(x) \quad \Rightarrow \quad g(x) = \tan(x)$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)\tan(x)\tan(x)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)\tan^2(x)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)(\sec^2(\mathrm{x})-1)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec^3(x)\,dx + \int \sec(x)\,dx$$

$$2\int \sec^3(x)\,dx = \tan(x)\sec(x) + \ln|\sec(x)+\tan(x)| + \mathrm{c}$$

$$\int \sec^3(x)\,dx = \frac{1}{2}[\tan(x)\sec(x) + \ln|\sec(x)+\tan(x)|] + \mathrm{c}$$

d)

$$\int \tan(\mathrm{x})dx = \int \frac{\mathrm{sen(x)}}{\cos(\mathrm{x})}dx$$

$$\int \tan(x)dx = -\int \frac{(\cos(x))'}{\cos(x)}dx$$

$$\int \tan(x)dx = -ln|\cos(x)| + c$$

f)

$$\int \text{cotan}(x)dx = \int \frac{\cos(x)}{\text{sen}(x)}dx$$

$$\int \text{cotan}(x)dx = \int \frac{(\text{sen}(x))'}{\text{sen}(x)}dx$$

$$\int \text{cotan}(x)dx = ln|\text{sen}(x)| + c$$

g)

$$\int \text{cosec}(x)\,dx$$

Multipliquemos por 1,

$$\int \text{cosec}(x)\frac{(\text{cosec}(x) + \text{cotan}(x))}{(\text{cosec}(x) + co\text{tan}(x))}dx$$

$$\int \frac{(\text{cosec}^2(x) + \text{cosec}(x)\,\text{cotan}(x))}{(\text{cosec}(x) + co\text{tan}(x))}dx = -\int \frac{(\text{cosec}(x) + co\text{tan}(x))'}{(\text{cosec}(x) + co\text{tan}(x))}dx$$

$$-\int \frac{(\text{cosec}(x) + co\text{tan}(x))'}{(\text{cosec}(x) + co\text{tan}(x))}dx = -\ln|\text{cosec}(x) + co\text{tan}(x)| + \text{c}$$

h)

$$\int \text{e}^{\text{x}} sen(2x)dx$$

$$f(x) = sen(2x) \quad \Rightarrow \quad f'(x) = \cos(2x)\cdot 2$$

$$g'(x) = e^x \quad \Rightarrow \quad g(x) = e^x$$

$$\int \text{e}^{\text{x}} sen(2x)dx = sen(2x)e^x - 2\int \text{e}^{\text{x}} cos(2x)dx$$

Otra vez por partes,

$$f(x) = cos(2x) \quad \Rightarrow \quad f'(x) = -sen(2x) \cdot 2$$

$$g'(x) = e^x \quad \Rightarrow \quad g(x) = e^x$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2\left[cos(2x)e^x + 2\int \mathrm{e}^{\mathrm{x}} sen(2x)dx \right]$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2cos(2x)e^x - 4\int \mathrm{e}^{\mathrm{x}} sen(2x)dx$$

$$5\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2cos(2x)e^x$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = \frac{1}{5}[sen(2x)e^x - 2cos(2x)e^x]$$

i)

$$\int \frac{e^x dx}{e^{2x}+1}$$

$$e^x = u \quad \Rightarrow \quad du = e^x dx \quad \Rightarrow \quad dx = \frac{du}{e^x}$$

$$\int \frac{e^x}{u^2+1}\frac{du}{e^x} = \int \frac{1}{u^2+1} du = \arctan(u) + c$$

$$\int \frac{e^x}{u^2+1}\frac{du}{e^x} = \int \frac{1}{u^2+1} du = \arctan(e^x) + c$$

j)

$$\int \frac{x}{\sqrt{x^2+1}} dx$$

$$1 + x^2 = u \quad \Rightarrow \quad du = 2xdx \quad \Rightarrow \quad dx = \frac{du}{2x}$$

$$\int \frac{x}{\sqrt{u}}\frac{du}{2x} = \frac{1}{2}\int u^{-\frac{1}{2}} du = \sqrt{u} + c = \sqrt{1+x^2} + c$$

k)

$$\int \frac{x}{\sqrt{1-x^4}}dx$$

$$x^2 = u \quad \Rightarrow \quad du = 2xdx \quad \Rightarrow \quad dx = \frac{du}{2x}$$

$$\int \frac{x}{\sqrt{1-u^2}}\frac{du}{2x} = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}du = \frac{1}{2}\arcsin{(u)} + c$$

$$\int \frac{x}{\sqrt{1-x^4}}dx == \frac{1}{2}\arcsin(x^2) + c$$

l)

$$\int \frac{1}{xln(x)}dx = \int \frac{\frac{1}{x}}{ln(x)}dx = ln\big|ln|x|\big| + c$$

m)

$$\int \frac{1}{4+x^2}dx = \int \frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)}dx$$

$$\int \frac{1}{4+x^2}dx = \frac{1}{2}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx$$

$$\frac{x}{2} = u \quad \Rightarrow \quad 2du = dx$$

$$\int \frac{1}{4+x^2}dx = \frac{1}{2}\int \frac{1}{1+u^2}2du$$

$$\int \frac{1}{4+x^2}dx = \arctan(u) + c$$

$$\int \frac{1}{4+x^2}dx = \arctan\left(\frac{x}{2}\right) + c$$

n)

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\int \frac{2x+1-1}{1+x+x^2}dx$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[\int \frac{2x+1}{1+x+x^2}dx - \int \frac{1}{1+x+x^2}dx\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \int \frac{1}{\frac{3}{4}+\left(x+\frac{1}{2}\right)^2}dx\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \int \frac{1}{\frac{3}{4}\left(1+\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)^2\right)}\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{4}{3}\int \frac{dx}{\left(1+\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)^2\right)}\right]$$

$$\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right] = u \quad \Rightarrow \quad du = \frac{2}{\sqrt{3}}dx$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{4}{3}\frac{\sqrt{3}}{2}\int \frac{du}{(1+u^2)}\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{2\sqrt{3}}{3}Arctan\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)\right] + c$$

Ñ)

$$\int \sqrt{1+4x^2}\,dx = \int \sqrt{1+(2x)^2}\,dx$$

$$2x = \tan(\theta) \quad \Rightarrow \quad dx = \frac{1}{2}\sec^2(\theta)\,d\theta$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{2}\int \sqrt{1+\tan(\theta)^2}\sec^2(\theta)\,d\theta$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{2}\int \sec^3(\theta)\,d\theta$$

$$\int \sec^3(\theta)\,dx = \int \sec^2(\theta)\sec(\theta)dx$$

$$f(x) = \sec(\theta) \quad \Rightarrow \quad f'(x) = \sec(\theta)\tan(\theta)$$

$$g'(x) = \sec^2(\theta) \quad \Rightarrow \quad g(\theta) = \tan(\theta)$$

$$\int \sec^3(\theta)\,d\theta = \tan(\theta)\sec(\theta) - \int \sec(\theta)\tan(\theta)\tan(\theta)d\theta$$

$$\int \sec^3(\theta)\,d\theta = \tan(\theta)\sec(x) - \int \sec(\theta)\tan^2(\theta)d\theta$$

$$\int \sec^3(\theta)\,d\theta = \tan(\theta)\sec(\theta) - \int \sec(\theta)\,(\sec^2(\theta) - 1)d\theta$$

$$\int \sec^3(\theta)\,d\theta = \tan(\theta)\sec(\theta) - \int \sec^3(\theta)\,d\theta + \int \sec(\theta)\,d\theta$$

$$2\int \sec^3(\theta)\,d\theta = \tan(\theta)\sec(\theta) + \ln|\sec(\theta) + \tan(\theta)| + c$$

$$\int \sec^3(\theta)\,d\theta = \frac{1}{2}[\tan(\theta)\sec(\theta) + \ln|\sec(\theta) + \tan(\theta)|] + c$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{2}\frac{1}{2}[\tan(\theta)\sec(\theta) + \ln|\sec(\theta) + \tan(\theta)|] + c$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{4}\left[2x\sqrt{1+4x^2} + \ln\left|\sqrt{1+4x^2} + 2x\right|\right] + c$$

o)

$$\int \frac{1}{1-\cos(\theta)}d\theta = \int \frac{1}{1-\cos(\theta)}\frac{1+\cos(\theta)}{1+\cos(\theta)}d\theta$$

$$\int \frac{1}{1-\cos(\theta)}d\theta = \int \frac{1+\cos(\theta)}{1-\cos^2(\theta)}d\theta$$

$$\int \frac{1}{1-\cos(\theta)}d\theta = \int \frac{1+\cos(\theta)}{\text{sen}^2(\theta)}d\theta$$

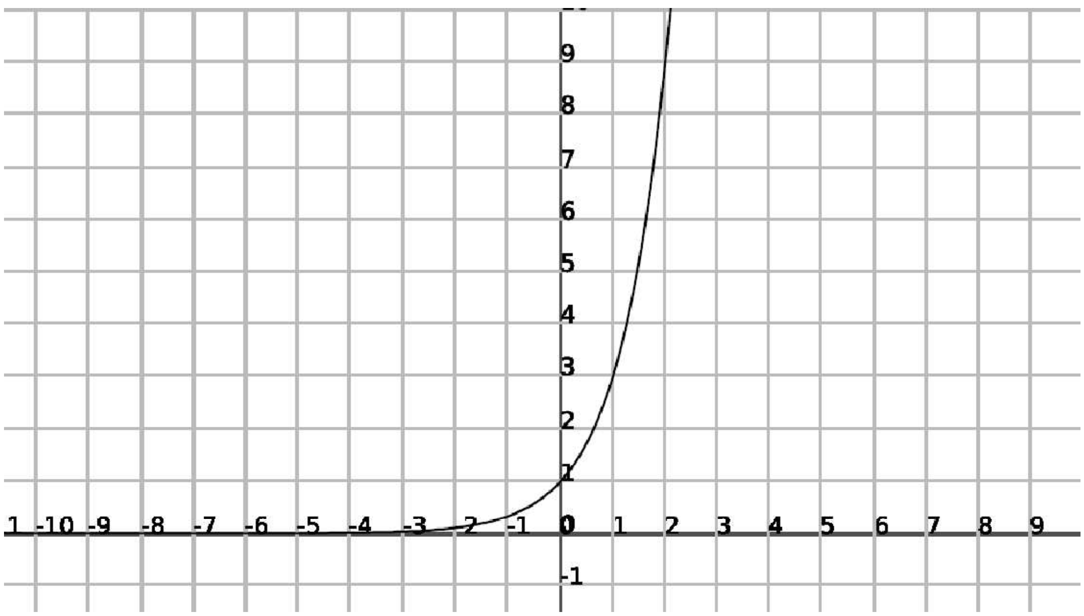
$$\int \frac{1}{1-\cos(\theta)}d\theta = \int \text{cosec}^2(\theta)d\theta + \int \text{cosec}(\theta)\text{cotan}(\theta)d\theta$$

$$\int \frac{1}{1-\cos(\theta)}d\theta = -cotan(\theta) - cosec(\theta) + c$$

4. Grafique las siguientes funciones:

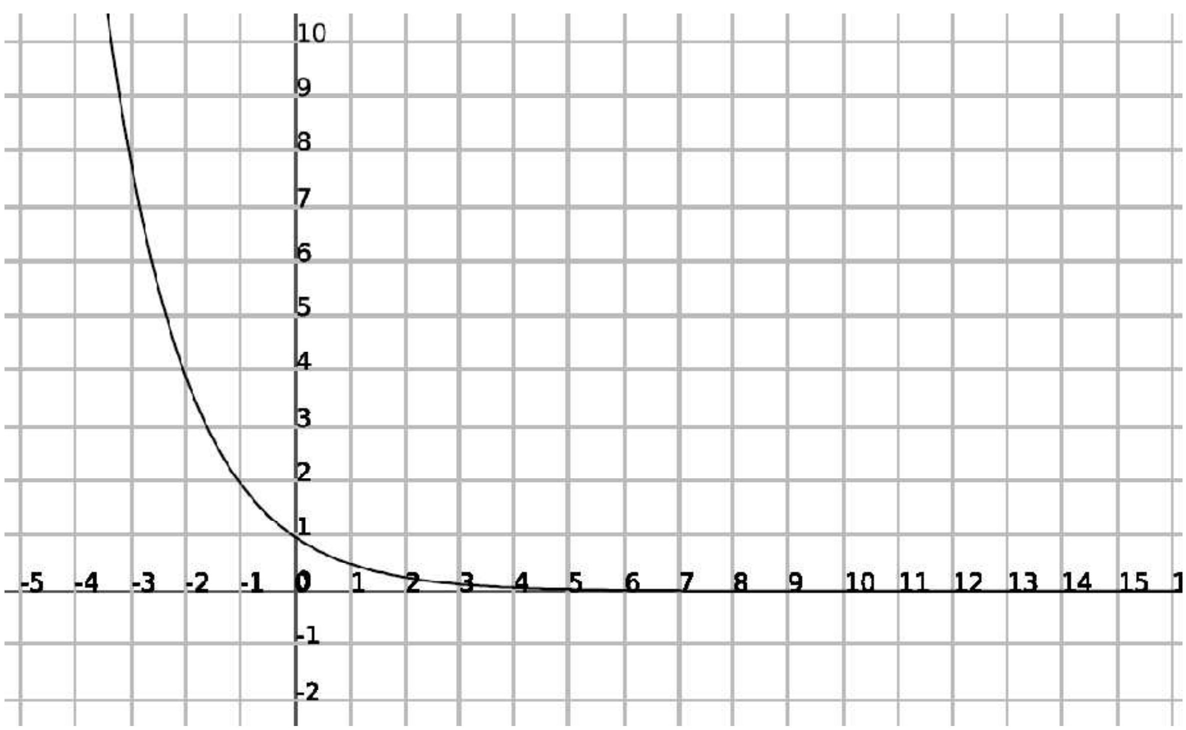
a)

$$f: R \rightarrow R \ definida \ por \ f(x) = 3^x$$



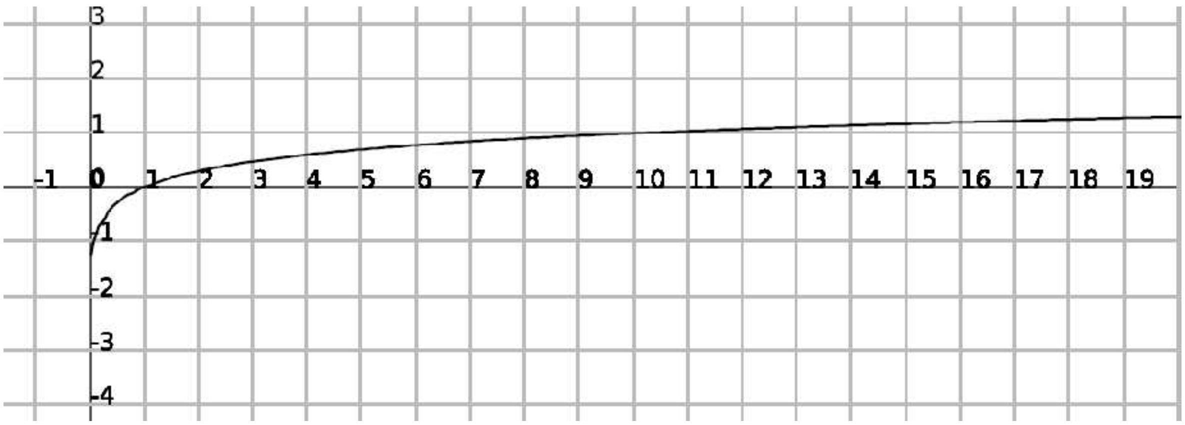
b)

$$f: R \rightarrow R \ definida \ por \ f(x) = 0{,}5^x$$



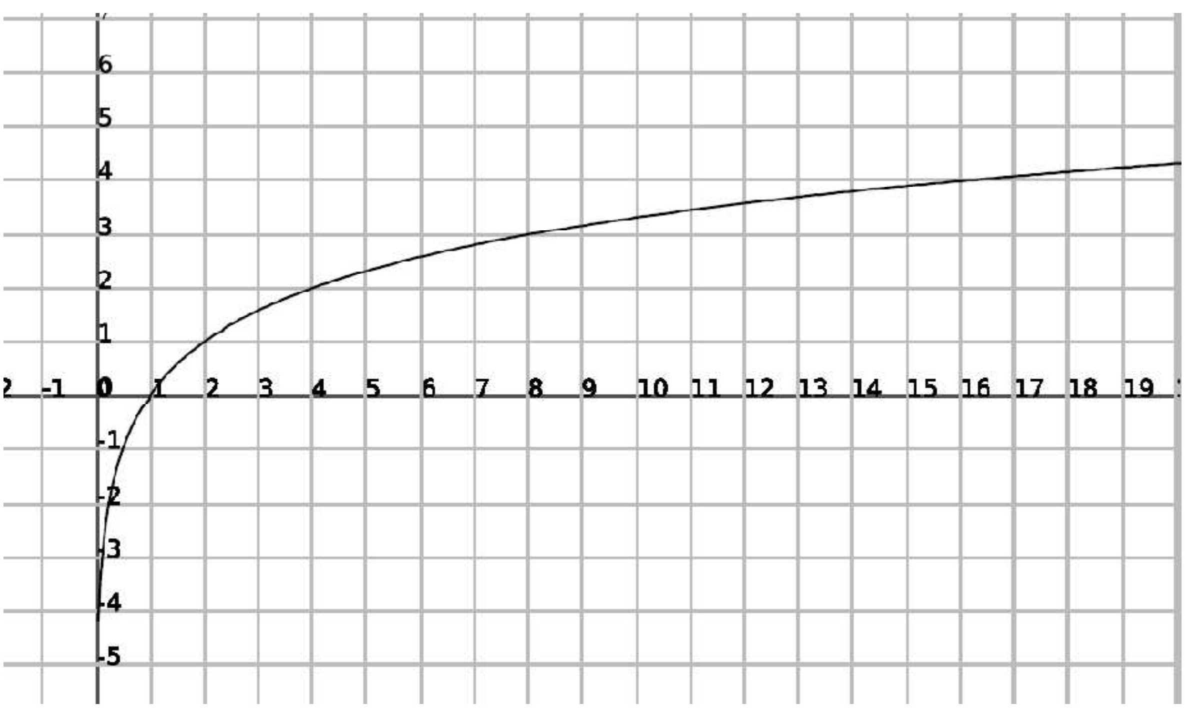
c)

$$f: R \to R \ definida \ por \ f(x) = log_{10}(x) = \frac{\ln(x)}{\ln(10)}$$



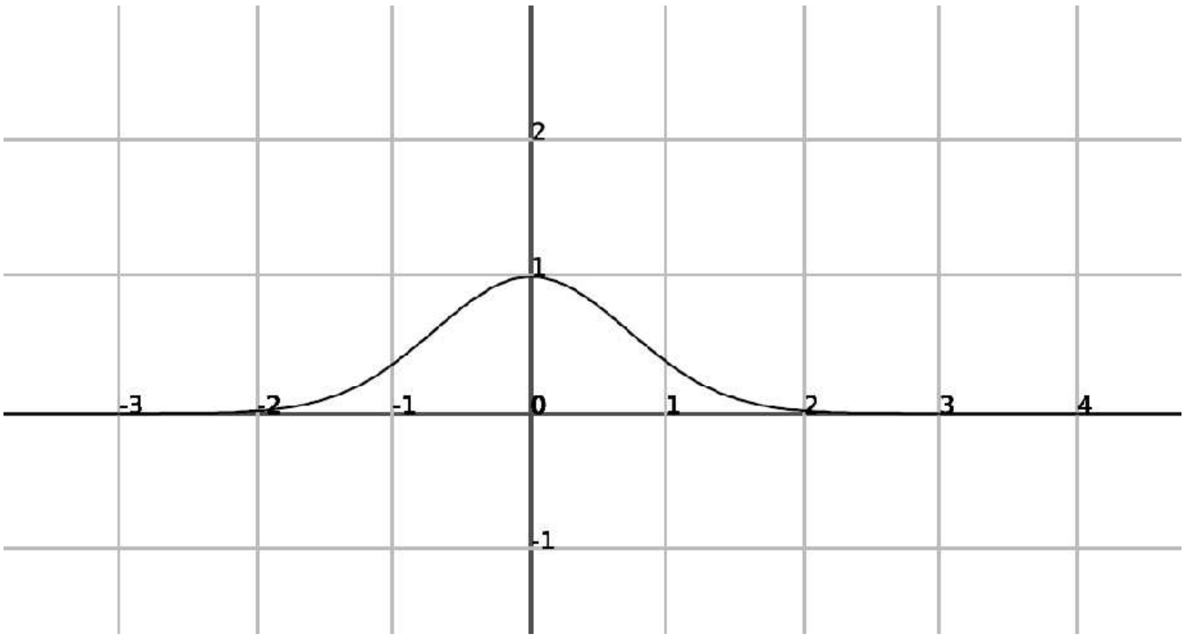
d)

$$f: R \to R \ definida \ por \ f(x) = log_2(x) = \frac{\ln(x)}{\ln(2)}$$



e)

$$f: R \to R \ definida\ por\ f(x) = e^{-x^2}$$



5. Las funciones cosh, senh : R $\to$ R definidas por $\cosh(x) = \frac{e^x+e^{-x}}{2}$ y $\mathrm{senh}(x) = \frac{e^x-e^{-x}}{2}$ se llaman el *coseno hiperbólico* y el *seno hiperbólico* respectivamente

a) Muestre que $\mathrm{senh}'(x) = \cosh(x)$ y que $\cosh'(x) = \mathrm{senh}(x)$

$$\mathrm{senh}'(x) = \left(\frac{e^x - e^{-x}}{2}\right)'$$

$$\mathrm{senh}'(x) = \frac{e^x + e^{-x}}{2}$$

$$\mathrm{senh}'(x) = \cosh(x)$$

$$\cosh'(x) = \left(\frac{e^x + e^{-x}}{2}\right)'$$

$$\cosh'(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh'(x) = senh(x)$$

b) Muestre que cosh es par y senh impar

$$\text{senh}(-x) = \left(\frac{e^{-x} - e^{x}}{2}\right)$$

$$\text{senh}(-x) = -\left(\frac{e^{x} - e^{-x}}{2}\right)$$

$$\text{senh}(-x) = -\text{senh}\ (x)$$

Luego, es impar.

$$\cosh(-x) = \frac{e^{-x} + e^{x}}{2}$$

$$\cosh(-x) = \cosh(x)$$

Luego, es par

c) Muestre que $\cosh^2(\mathrm{x}) - \text{senh}^2(\mathrm{x}) = 1$

$$\cosh^2(\mathrm{x}) - \text{senh}^2(\mathrm{x}) = \left(\frac{e^{x} + e^{-x}}{2}\right)^2 - \left(\frac{e^{x} - e^{-x}}{2}\right)^2$$

$$\cosh^2(\mathrm{x}) - \text{senh}^2(\mathrm{x}) = \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4}$$

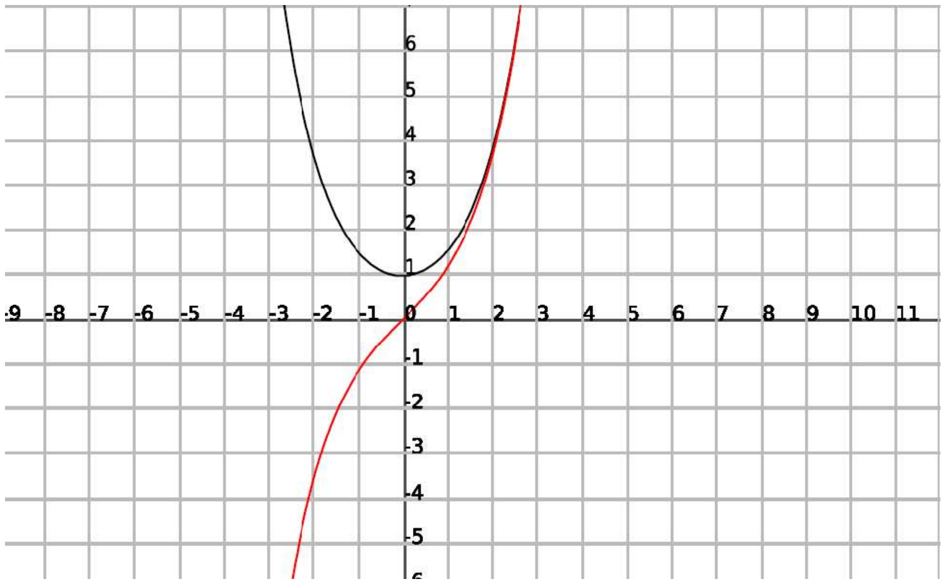
$$\cosh^2(\mathrm{x}) - \text{senh}^2(\mathrm{x}) = \frac{2+2}{4}$$

$$\cosh^2(\mathrm{x}) - \text{senh}^2(\mathrm{x}) = 1$$

d) Grafique cosh(x) y senh(x)

$$f: R \rightarrow R\ definida\ por\ f(x) = \cosh\ (x)$$

$$f: R \rightarrow R\ definida\ por\ f(x) = senh(x)$$



6. . ¿Es cierto que para cualquier $x \geq 0$ se tiene que $e^x - 1 \leq xe^x$?

7. ¿Es cierto que para cualquier $x \geq 0$ se tiene que $e^x - 1 \leq xe^x$?

8. Caulcula las siguientes integrales:

a)

$$\int \frac{1}{(1+x^2)^2}\,dx$$

$$x = \tan(\theta) \quad \Rightarrow \quad dx = \sec^2(\theta)\,d\theta$$

$$\int \frac{1}{(1+x^2)^2}\,dx = \int \frac{1}{(1+\tan(\theta)^2)^2}\sec^2(\theta)\,d\theta$$

$$\int \frac{1}{(1+x^2)^2}\,dx = \int \frac{1}{(\sec^2(\theta))^2}\sec^2(\theta)\,d\theta$$

$$\int \frac{1}{(1+x^2)^2}\,dx = \int \sec^{-2}(\theta)\,d\theta$$

$$\int \frac{1}{(1+x^2)^2}\,dx = \int \cos^2(\theta)\,d\theta$$

$$\int \frac{1}{(1+x^2)^2}dx = \int \left(\frac{1+\cos(2\theta)}{2}\right)d\theta$$

$$\int \frac{1}{(1+x^2)^2}dx = \frac{\theta}{2} + \frac{sen(2\theta)}{4} + c$$

$$\int \frac{1}{(1+x^2)^2}dx = \frac{Arctan(x)}{2} + \frac{sen(2\theta)}{4} + c$$

$$\int \frac{1}{(1+x^2)^2}dx = \frac{Arctan(x)}{2} + \frac{sen(\theta)\cos(\theta)}{2} + c$$

$$\int \frac{1}{(1+x^2)^2}dx = \frac{Arctan(x)}{2} + \frac{1}{2}\left(\frac{x}{1+x^2}\right) + c$$

b)

$$\int \frac{x+3}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2x+6}{x^2+2x+2}dx$$

$$\int \frac{x+3}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2x+2+4}{x^2+2x+2}dx$$

$$\int \frac{x+3}{x^2+2x+2}dx = \frac{1}{2}\left[\int \frac{2x+2}{x^2+2x+2}dx + \int \frac{4}{x^2+2x+2}d\right]$$

$$\int \frac{x+3}{x^2+2x+2}dx = \frac{1}{2}\left[ln|x^2+2x+2| + \int \frac{4}{(x+1)^2+1}d\right]$$

$$\int \frac{x+3}{x^2+2x+2}dx = \frac{1}{2}[ln|x^2+2x+2| + 4Arctan(x+1)] + c$$

c)

$$\int \frac{1+e^x}{1-e^x}dx$$

$$e^x = u \quad \Rightarrow \quad e^x dx = du \quad \Rightarrow \quad dx = \frac{du}{u}$$

$$\int \frac{1+e^x}{1-e^x}dx = \int \frac{1+u}{(1-u)u}du$$

$$\frac{1+u}{(1-u)u} = \frac{A}{1-u} + \frac{B}{u}$$

$$\frac{1+u}{(1-u)u} = \frac{Au + B(1-u)}{(1-u)u}$$

$$\frac{1+u}{(1-u)u}=\frac{(A-B)u+B}{(1-u)u}$$

$$B=1$$

$$A-B=1 \quad \Rightarrow \quad A=2$$

$$\int \frac{1+u}{(1-u)u}du=\int \frac{2du}{1-u}+\int \frac{du}{u}$$

$$\int \frac{1+u}{(1-u)u}du=-2ln|1-u|+ln|u|+c$$

$$\int \frac{1+u}{(1-u)u}du=-2ln|1-e^x|+ln|e^x|+c$$

$$\int \frac{1+u}{(1-u)u}du=-2ln|1-e^x|+x+c$$

d)

$$\int \frac{8x^2+6x+4}{x+1}dx=\int \frac{(x+1)(8x-2)+6}{x+1}dx$$

$$\int \frac{8x^2+6x+4}{x+1}dx=\int (8x-2)dx+\int \frac{6}{x+1}dx$$

$$\int \frac{8x^2+6x+4}{x+1}dx=\frac{8x^2}{2}-2x+6\cdot ln|x+1|+c$$

e)

$$\int \frac{2x+1}{x^3-3x^2+3x-1}dx=\int \frac{2x+1}{(x-1)^3}dx$$

$$\frac{2x+1}{(x-1)^3}=\frac{A}{(x-1)^1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

$$\frac{2x+1}{(x-1)^3}=\frac{C+B(x-1)+A(x-1)^2}{(x-1)^3}$$

$$\frac{2x+1}{(x-1)^3}=\frac{Ax^2+(B-2A)x+C-B+A}{(x-1)^3}$$

$$A=0$$

$$B-2A=2 \quad \Rightarrow \quad B=2$$

$$C-B+A=1 \quad \Rightarrow \quad C=3$$

$$\int \frac{2x+1dx}{(x-1)^3} = \int \frac{0dx}{(x-1)^1} + \int \frac{2dx}{(x-1)^2} + \int \frac{3dx}{(x-1)^3}$$

$$\int \frac{2x+1dx}{(x-1)^3} = \int \frac{2dx}{(x-1)^2} + \int \frac{3dx}{(x-1)^3}$$

$$\int \frac{2x+1dx}{(x-1)^3} = \frac{-2}{(x-1)^1} + \frac{-3}{2(x-1)^2} + c$$

f)

$$\int \frac{x^3+x+2}{x^4+2x^2+1}dx = \int \frac{x^3+x+2}{(x^2+1)^2}dx$$

$$\frac{x^3+x+2}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)^1} + \frac{Cx+D}{(x^2+1)^2}$$

$$\frac{x^3+x+2}{(x^2+1)^2} = \frac{Cx+D+(x^2+1)(Ax+B)}{(x^2+1)^2}$$

$$\frac{x^3+x+2}{(x^2+1)^2} = \frac{Ax^3+Bx^2+(C+A)x+D+B}{(x^2+1)^2}$$

$$A = 1$$

$$B = 0$$

$$C + A = 1 \quad \Rightarrow \quad C = 0$$

$$D + B = 2 \quad \Rightarrow \quad D = 2$$

$$\int \frac{x^3+x+2}{(x^2+1)^2}dx = \int \frac{xdx}{x^2+1} + \int \frac{2dx}{(x^2+1)^2}$$

$$\int \frac{x^3+x+2}{(x^2+1)^2}dx = \frac{1}{2}\int \frac{2xdx}{x^2+1} + 2\int \frac{dx}{(x^2+1)^2}$$

$$\int \frac{x^3+x+2}{(x^2+1)^2}dx = \frac{1}{2}ln|x^2+1| + Arctan(x) + \left(\frac{x}{1+x^2}\right) + c$$

g)

$$\int \frac{1}{x^4+1}dx = \int \frac{1}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}dx$$

$$\frac{1}{x^4+1} = \frac{Ax+B}{(x^2+\sqrt{2}x+1)} + \frac{Cx+D}{(x^2-\sqrt{2}x+1)}$$

$$\frac{1}{x^4+1} = \frac{(Ax+B)(x^2-\sqrt{2}x+1)+(Cx+D)(x^2+\sqrt{2}x+1)}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}$$

$$\frac{1}{x^4+1} = \frac{x^3(A+C)+x^2(B+D+\sqrt{2}C-\sqrt{2}A)+x(A+C+\sqrt{2}D-\sqrt{2}B)+\ B+D}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}$$

$$A+C=0 \quad \Rightarrow \quad C=-A$$

$$B+D+\sqrt{2}C-\sqrt{2}A=0 \quad \Rightarrow \quad B+D+2\sqrt{2}C=0 \quad \Rightarrow \quad C=\frac{-1}{2\sqrt{2}}=-A$$

$$A+C+\sqrt{2}D-\sqrt{2}B=0 \quad \Rightarrow \quad D=B$$

$$B+D=1 \quad \Rightarrow \quad D=B=\frac{1}{2}$$

$$\frac{1}{x^4+1} = \frac{\frac{1}{2\sqrt{2}}x+\frac{1}{2}}{(x^2+\sqrt{2}x+1)} + \frac{\frac{-1}{2\sqrt{2}}x+\frac{1}{2}}{(x^2-\sqrt{2}x+1)}$$

$$\int \frac{1}{x^4+1}dx = \frac{1}{2\sqrt{2}}\left[\int \frac{x+\sqrt{2}}{(x^2+\sqrt{2}x+1)}dx - \int \frac{x-\sqrt{2}}{(x^2-\sqrt{2}x+1)}dx\right]$$

$$\int \frac{1}{x^4+1}dx = \frac{1}{4\sqrt{2}}\left[\int \frac{2x+\sqrt{2}+\sqrt{2}}{(x^2+\sqrt{2}x+1)}dx - \int \frac{2x-\sqrt{2}-\sqrt{2}}{(x^2-\sqrt{2}x+1)}dx\right]$$

$$\int \frac{dx}{x^4+1} = \frac{1}{4\sqrt{2}}\left[\int \frac{(2x+\sqrt{2})dx}{(x^2+\sqrt{2}x+1)} + \int \frac{\sqrt{2}dx}{(x^2+\sqrt{2}x+1)} - \int \frac{(2x-\sqrt{2})dx}{(x^2-\sqrt{2}x+1)} + \int \frac{\sqrt{2}dx}{(x^2-\sqrt{2}x+1)}\right]$$

$$\int \frac{dx}{x^4+1} = \frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\left(x^2+\sqrt{2}x+1\right)} - ln\left|x^2-\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\left(x^2-\sqrt{2}x+1\right)}\right]$$

$$\int \frac{dx}{x^4+1} = \frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\left(x+\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}} - ln\left|x^2-\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\left(x-\frac{\sqrt{2}}{2}\right)^2+\frac{1}{2}}\right]$$

$$\int \frac{dx}{x^4+1} = \frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\frac{1}{2}\left[\left(\frac{\sqrt{2}}{1}\left(x+\frac{\sqrt{2}}{2}\right)\right)^2+1\right]} - ln\left|x^2-\sqrt{2}x+1\right| + \int \frac{\sqrt{2}dx}{\frac{1}{2}\left[\left(\frac{\sqrt{2}}{1}\left(x-\frac{\sqrt{2}}{2}\right)\right)^2+1\right]}\right]$$

$$\int \frac{dx}{x^4+1} = \frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right| + \int \frac{2\sqrt{2}dx}{\left[\left(\frac{\sqrt{2}}{1}\left(x+\frac{\sqrt{2}}{2}\right)\right)^2+1\right]} - ln\left|x^2-\sqrt{2}x+1\right| + \int \frac{2\sqrt{2}dx}{\left[\left(\frac{\sqrt{2}}{1}\left(x-\frac{\sqrt{2}}{2}\right)\right)^2+1\right]}\right]$$

$$\frac{\sqrt{2}}{1}\left(x+\frac{\sqrt{2}}{2}\right)=u \quad \Rightarrow \quad du=\sqrt{2}dx$$

$$\frac{\sqrt{2}}{1}\left(x-\frac{\sqrt{2}}{2}\right)=m \quad \Rightarrow \quad dm=\sqrt{2}dx$$

$$\int\frac{dx}{x^4+1}=\frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right|+\int\frac{2\sqrt{2}}{[u^2+1]}\frac{du}{\sqrt{2}}-ln\left|x^2-\sqrt{2}x+1\right|+\int\frac{2\sqrt{2}}{[m^2+1]}\frac{dm}{\sqrt{2}}\right]$$

$$\int\frac{dx}{x^4+1}=\frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right|+2Arctan(u)-ln\left|x^2-\sqrt{2}x+1\right|+2Arctan(m)+c\right]$$

$$\int\frac{dx}{x^4+1}=\frac{1}{4\sqrt{2}}\left[ln\left|x^2+\sqrt{2}x+1\right|+2Arctan\left(\frac{\sqrt{2}}{1}\left(x+\frac{\sqrt{2}}{2}\right)\right)-ln\left|x^2-\sqrt{2}x+1\right|+2Arctan\left(\frac{\sqrt{2}}{1}\left(x-\frac{\sqrt{2}}{2}\right)\right)\right]+c$$

h)

$$\int\frac{2x+1}{x^3+5x^2+6x}dx=\int\frac{2x+1}{x(x^2+5x+6)}dx$$

$$\int\frac{2x+1}{x^3+5x^2+6x}dx=\int\frac{2x+1}{x(x+3)(x+2)}dx$$

$$\frac{2x+1}{x(x+3)(x+2)}=\frac{A}{x}+\frac{B}{(x+3)}+\frac{C}{(x+2)}$$

$$\frac{2x+1}{x(x+3)(x+2)}=\frac{A(x+3)(x+2)+Bx(x+2)+Cx(x+3)}{x(x+3)(x+2)}$$

$$\frac{2x+1}{x(x+3)(x+2)}==\frac{(A+B+C)x^2+(5A+2B+3C)x+6A}{x(x+3)(x+2)}$$

$$A=\frac{1}{6}$$

$$\frac{1}{6}+B+C=0 \quad \Rightarrow \quad C=-\frac{1}{6}-B$$

$$\frac{5}{6}+2B+3C=2 \quad \Rightarrow \quad \frac{5}{6}+2B+3\left(-\frac{1}{6}-B\right)=2$$

$$B=-2+\frac{5}{6}-\frac{1}{2}$$

$$B=\frac{-5}{3}$$

$$C=-\frac{1}{6}+\frac{5}{3}$$

$$C=\frac{3}{2}$$

$$\int \frac{2x+1}{x(x+3)(x+2)}dx=\int \quad \frac{dx}{6x}-\int \frac{5dx}{3(x+3)}+\int \frac{3dx}{2(x+2)}$$

$$\int \frac{2x+1}{x(x+3)(x+2)}dx=6ln|x|-\frac{5}{3}ln|x+3|+\frac{1}{2}ln|x+2|+c$$

i)

$$\int \frac{x^3+x}{x^2-1}dx=\int \frac{(x^2-1)x+2x}{x^2-1}dx$$

$$\int \frac{x^3+x}{x^2-1}dx=\int xdx+\int \frac{2x}{x^2-1}dx$$

$$\int \frac{x^3+x}{x^2-1}dx=\frac{x^2}{2}-2x+ln|x^2-1|+c$$