Guía 4 Matemática II Resuelta

Programa Académico de Bachillerato

1. Calcule los siguientes límites:

a)

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

Primero,

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x - 0}{1} = \frac{1}{1}$$

b)

$$\lim_{x \to 0} \frac{ln(1 + x)}{x}$$

Primero,

$$\lim_{x \to 0} \frac{ln(1 + x)}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x \to 0} \frac{\frac{1}{1 + x}}{1} = 1$$

c)

$$\lim_{x \to 0} \frac{a^x - 1}{x}$$

Primero,

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x\to 0}\frac{a^x-1}{x}=\lim_{x\to 0}\frac{a^x\ln(a)-0}{1}=\frac{1\ln(a)}{1}=\ln(a)$$

d)

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

Primero,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\lim_{x\to 0}\frac{a^x\ln(a)-b^x\ln(b)}{1}=\ln(a)-\ln(b)$$

e)

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}$$

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}=\lim_{x\to 0}\frac{\frac{ln(1+x)}{\ln(a)}}{x}$$

$$\lim_{x\to 0}\frac{log_a(1+x)}{x}=\frac{1}{\ln(a)}\lim_{x\to 0}\frac{ln(1+x)}{x}$$

Primero,

$$\frac{1}{\ln(a)}\lim_{x\to 0}\frac{ln(1+x)}{x}=\frac{0}{0}$$

Se puede aplicar la regla del L' Hopital,

$$\frac{1}{\ln(a)}\lim_{x\to 0}\frac{\frac{1}{1+x}}{1}=\frac{1}{\ln(a)}$$

2. Calcule las siguientes primitivas:

a)

$$\int \ln(x)\, dx$$

$$f(x) = \ln(x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

$$g'(x) = 1 \quad \Rightarrow \quad g(x) = x$$

$$\int \ln(x) \cdot 1dx = xln(x) - \int \frac{1}{x} xdx$$

$$\int \ln(x)\, dx = xln(x) - x + c$$

b)

$$\int \text{Arctg}(x)\, dx$$

$$f(x) = \text{Arctg}(x) \quad \Rightarrow \quad f'(x) = \frac{1}{x^2+1}$$

$$g'(x) = 1 \quad \Rightarrow \quad g(x) = x$$

$$\int \text{Arctg}(x) \cdot 1dx = x\, \text{Arctg}(x) - \int \frac{1}{x^2+1} xdx$$

$$\int \text{Arctg}(x) \cdot 1dx = x\, \text{Arctg}(x) - \int \frac{x}{x^2+1} dx$$

$$\int \text{Arctg}(x)\, dx = x\, \text{Arctg}(x) - \frac{1}{2}\int \frac{2x}{x^2+1} dx$$

$$\int \text{Arctg}(x)\, dx = x\, \text{Arctg}(x) - \frac{1}{2} ln|x^2+1| + c$$

c)

$$\int \sec(x)\,dx$$

Multipliquemos por 1,

$$\int \sec(x)\frac{(\sec(x)+\tan(x))}{(\sec(x)+\tan(x))}dx$$

$$\int \frac{(\sec^2(x)+\sec(x)\tan(x))}{(\sec(x)+\tan(x))}dx = \int \frac{(\sec(x)+\tan(x))'}{(\sec(x)+\tan(x))}dx$$

$$\int \frac{(\sec(x)+\tan(x))'}{(\sec(x)+\tan(x))}dx = \ln|\sec(x)+\tan(x)| + \mathrm{c}$$

d)

$$\int \sec^3(x)\,dx = \int \sec^2(x)\sec(x)dx$$

$$f(x) = \sec(x) \quad \Rightarrow \quad f'(x) = \sec(x)\tan(x)$$

$$g'(x) = \sec^2(x) \quad \Rightarrow \quad g(x) = \tan(x)$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)\tan(x)\tan(x)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)\tan^2(x)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec(x)(\sec^2(\mathrm{x})-1)dx$$

$$\int \sec^3(x)\,dx = \tan(x)\sec(x) - \int \sec^3(x)\,dx + \int \sec(x)\,dx$$

$$2\int \sec^3(x)\,dx = \tan(x)\sec(x) + \ln|\sec(x)+\tan(x)| + \mathrm{c}$$

$$\int \sec^3(x)\,dx = \frac{1}{2}[\tan(x)\sec(x) + \ln|\sec(x)+\tan(x)|] + \mathrm{c}$$

d)

$$\int \tan(\mathrm{x})dx = \int \frac{\mathrm{sen(x)}}{\cos(\mathrm{x})}dx$$

$$\int \tan(\mathrm{x})dx = -\int \frac{(\cos(\mathrm{x}))'}{\cos(\mathrm{x})}dx$$

$$\int \tan(\mathrm{x})dx = -ln|\cos(x)| + c$$

f)

$$\int \mathrm{cotan}(\mathrm{x})dx = \int \frac{\cos(\mathrm{x})}{\mathrm{sen}(\mathrm{x})}dx$$

$$\int \mathrm{cotan}(\mathrm{x})dx = \int \frac{(\mathrm{sen}(\mathrm{x}))'}{\mathrm{sen}(\mathrm{x})}dx$$

$$\int \mathrm{cotan}(\mathrm{x})dx = ln|\mathrm{sen}(x)| + c$$

g)

$$\int \mathrm{cosec}(x)\,dx$$

Multipliquemos por 1,

$$\int \mathrm{cosec}(x)\frac{(\mathrm{cosec}(x) + \mathrm{cotan}(x))}{(\mathrm{cosec}(x) + co\mathrm{tan}(x))}dx$$

$$\int \frac{(\mathrm{cosec}^2(x) + \mathrm{cosec}(x)\,\mathrm{cotan}(x))}{(\mathrm{cosec}(x) + co\mathrm{tan}(x))}dx = -\int \frac{(\mathrm{cosec}(x) + co\mathrm{tan}(x))'}{(\mathrm{cosec}(x) + co\mathrm{tan}(x))}dx$$

$$-\int \frac{(\mathrm{cosec}(x) + co\mathrm{tan}(x))'}{(\mathrm{cosec}(x) + co\mathrm{tan}(x))}dx = -\ln|\mathrm{cosec}(x) + co\mathrm{tan}(x)| + \mathrm{c}$$

h)

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx$$

$$f(x) = sen(2x) \quad \Rightarrow \quad f'(x) = \cos(2x)\cdot 2$$

$$g'(x) = e^x \quad \Rightarrow \quad g(x) = e^x$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2\int \mathrm{e}^{\mathrm{x}} cos(2x)dx$$

Otra vez por partes,

$$f(x) = cos(2x) \quad \Rightarrow \quad f'(x) = -sen(2x) \cdot 2$$

$$g'(x) = e^x \quad \Rightarrow \quad g(x) = e^x$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2\left[cos(2x)e^x + 2\int \mathrm{e}^{\mathrm{x}} sen(2x)dx \right]$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2cos(2x)e^x - 4\int \mathrm{e}^{\mathrm{x}} sen(2x)dx$$

$$5\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = sen(2x)e^x - 2cos(2x)e^x$$

$$\int \mathrm{e}^{\mathrm{x}} sen(2x)dx = \frac{1}{5}[sen(2x)e^x - 2cos(2x)e^x]$$

i)

$$\int \frac{e^x dx}{e^{2x} + 1}$$

$$e^x = u \quad \Rightarrow \quad du = e^x dx \quad \Rightarrow \quad dx = \frac{du}{e^x}$$

$$\int \frac{e^x}{u^2 + 1}\frac{du}{e^x} = \int \frac{1}{u^2 + 1}du = \arctan(u) + c$$

$$\int \frac{e^x}{u^2 + 1}\frac{du}{e^x} = \int \frac{1}{u^2 + 1}du = \arctan(e^x) + c$$

j)

$$\int \frac{x}{\sqrt{x^2 + 1}}dx$$

$$1 + x^2 = u \quad \Rightarrow \quad du = 2xdx \quad \Rightarrow \quad dx = \frac{du}{2x}$$

$$\int \frac{x}{\sqrt{u}}\frac{du}{2x} = \frac{1}{2}\int u^{-\frac{1}{2}}du = \sqrt{u} + c = \sqrt{1 + x^2} + c$$

k)

$$\int \frac{x}{\sqrt{1-x^4}}dx$$

$$x^2 = u \quad \Rightarrow \quad du = 2xdx \quad \Rightarrow \quad dx = \frac{du}{2x}$$

$$\int \frac{x}{\sqrt{1-u^2}}\frac{du}{2x} = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}du = \frac{1}{2}\arcsin{(u)} + c$$

$$\int \frac{x}{\sqrt{1-x^4}}dx == \frac{1}{2}\arcsin(x^2) + c$$

l)

$$\int \frac{1}{xln(x)}dx = \int \frac{\frac{1}{x}}{ln(x)}dx = ln\big|ln|x|\big| + c$$

m)

$$\int \frac{1}{4+x^2}dx = \int \frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)}dx$$

$$\int \frac{1}{4+x^2}dx = \frac{1}{2}\int \frac{1}{1+\left(\frac{x}{2}\right)^2}dx$$

$$\frac{x}{2} = u \quad \Rightarrow \quad 2du = dx$$

$$\int \frac{1}{4+x^2}dx = \frac{1}{2}\int \frac{1}{1+u^2}2du$$

$$\int \frac{1}{4+x^2}dx = \arctan(u) + c$$

$$\int \frac{1}{4+x^2}dx = \arctan\left(\frac{x}{2}\right) + c$$

n)

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\int \frac{2x+1-1}{1+x+x^2}dx$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[\int \frac{2x+1}{1+x+x^2}dx - \int \frac{1}{1+x+x^2}dx\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \int \frac{1}{\frac{3}{4}+\left(x+\frac{1}{2}\right)^2}dx\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \int \frac{1}{\frac{3}{4}\left(1+\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)^2\right)}\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{4}{3}\int \frac{dx}{\left(1+\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)^2\right)}\right]$$

$$\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right] = u \quad \Rightarrow \quad du = \frac{2}{\sqrt{3}}dx$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{4}{3}\frac{\sqrt{3}}{2}\int \frac{du}{(1+u^2)}\right]$$

$$\int \frac{x}{1+x+x^2}dx = \frac{1}{2}\left[ln(1+x+x^2) - \frac{2\sqrt{3}}{3}Arctan\left(\frac{2}{\sqrt{3}}\left[x+\frac{1}{2}\right]\right)\right] + c$$

Ñ)

$$\int \sqrt{1+4x^2}\,dx = \int \sqrt{1+(2x)^2}\,dx$$

$$2x = \tan(\theta) \quad \Rightarrow \quad dx = \frac{1}{2}\sec^2(\theta)\,d\theta$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{2}\int \sqrt{1+\tan(\theta)^2}\sec^2(\theta)\,d\theta$$

$$\int \sqrt{1+4x^2}\,dx = \frac{1}{2}\int \sec^3(\theta)\,d\theta$$

$$\int \sec^3(\theta)\, dx = \int \sec^2(\theta) \sec(\theta) dx$$

$$f(x) = \sec(\theta) \quad \Rightarrow \quad f'(x) = \sec(\theta) \tan(\theta)$$

$$g'(x) = \sec^2(\theta) \quad \Rightarrow \quad g(\theta) = \tan(\theta)$$

$$\int \sec^3(\theta)\, d\theta = \tan(\theta)\sec(\theta) - \int \sec(\theta) \tan(\theta)\tan(\theta) d\theta$$

$$\int \sec^3(\theta)\, d\theta = \tan(\theta)\sec(x) - \int \sec(\theta) \tan^2(\theta) d\theta$$

$$\int \sec^3(\theta)\, d\theta = \tan(\theta)\sec(\theta) - \int \sec(\theta) (\sec^2(\theta) - 1) d\theta$$

$$\int \sec^3(\theta)\, d\theta = \tan(\theta) \sec(\theta) - \int \sec^3(\theta)\, d\theta + \int \sec(\theta)\, d\theta$$

$$2\int \sec^3(\theta)\, d\theta = \tan(\theta) \sec(\theta) + \ln|\sec(\theta) + \tan(\theta)| + c$$

$$\int \sec^3(\theta)\, d\theta = \frac{1}{2}[\tan(\theta) \sec(\theta) + \ln|\sec(\theta) + \tan(\theta)|] + c$$

$$\int \sqrt{1+4x^2}\, dx = \frac{1}{2}\frac{1}{2}[\tan(\theta) \sec(\theta) + \ln|\sec(\theta) + \tan(\theta)|] + c$$

$$\int \sqrt{1+4x^2}\, dx = \frac{1}{4}\left[2x\sqrt{1+4x^2} + \ln\left|\sqrt{1+4x^2} + 2x\right|\right] + c$$

o)

$$\int \frac{1}{1-\cos(\theta)} d\theta = \int \frac{1}{1-\cos(\theta)} \frac{1+\cos(\theta)}{1+\cos(\theta)} d\theta$$

$$\int \frac{1}{1-\cos(\theta)} d\theta = \int \frac{1+\cos(\theta)}{1-\cos^2(\theta)} d\theta$$

$$\int \frac{1}{1-\cos(\theta)} d\theta = \int \frac{1+\cos(\theta)}{\operatorname{sen}^2(\theta)} d\theta$$

$$\int \frac{1}{1-\cos(\theta)} d\theta = \int \operatorname{cosec}^2(\theta) d\theta + \int \operatorname{cosec}(\theta)\operatorname{cotan}(\theta) d\theta$$

$$\int \frac{1}{1-\cos(\theta)} d\theta = -cotan(\theta)$$