Ejercicio 9.6. $t = x + \sqrt{x^2 \pm a^2} \overset{*}{\Rightarrow} dt = \left(1 + \dfrac{x}{\sqrt{x^2 \pm a^2}}\right) dx$

$t - x = \sqrt{x^2 \pm a^2}$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \int \frac{1}{t}\, dt = \ln|t| + c$$

$$= \ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$$

$$dt = \frac{\sqrt{x^2 \pm a^2} + x}{\sqrt{x^2 \pm a^2}}\, dx$$

$$dt = \frac{t}{\sqrt{x^2 \pm a^2}}\, dx$$

$$\underbrace{\frac{1}{t}\, dt = \frac{dx}{\sqrt{x^2 \pm a^2}}}_{\text{propuesta}}$$

desarrollo en inversa para hallar t.

$*$

$$\int \left(1 + \frac{x}{\sqrt{x^2 \pm a^2}}\right) dx = x + \int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx = \int \frac{du}{2\sqrt{u}} = \sqrt{u} + c$$

$$= \sqrt{x^2 \pm a^2} + c$$

$u = x^2 \pm a^2$

$du = 2x\, dx$

$\dfrac{du}{2} = x\, dx$

$$I = \int x^2 \sqrt{x - 7}\, dx = \int (t^2+7)^2\, t \cdot 2t\, dt = \int 2(t^4 + 14t^2 + 49)\, t^2\, dt$$

$$= 2\int (t^6 + 14t^4 + 49t^2)\, dt = 2\left(\frac{t^7}{7} + \frac{14}{5}t^5 + \frac{49}{3}t^3\right)$$

$$= 2\frac{t^7}{7} + \frac{28}{5}t^5 + \frac{98}{3}t^3 + c$$

$$= \frac{30t^7 + 588t^5 + 3430t^3}{105} + c$$

$$= 2\left(16\left(\sqrt{x-7}\right)^7\right.$$

debo buscar hacer un reemplazo para eliminar la raiz cuadrada

propuesta: $t = \sqrt{x - 7}$

$t^2 = x - 7$

$t^2 + 7 = x$

$dx = 2t\, dt$

$$I = \int e^{ax}\sin(bx)\,dx = \sin(bx)\frac{e^{ax}}{a} - \int \frac{e^{ax}}{a}\, b\cos(bx)\,dx$$

$u_1 = \sin(bx) \Rightarrow du_1 = b\cos(bx)\,dx$

$dv_1 = e^{ax}\,dx$

$v_1 = \dfrac{e^{ax}}{a}$

$$= \frac{e^{ax}}{a}\sin(bx) - \frac{b}{a}\int e^{ax}\cos(bx)\,dx$$

$u_2 = \cos(bx) \rightarrow du = -b\sin(bx)\,dx$

$dv_1 = e^{ax} \Rightarrow v_1 = \dfrac{e^{ax}}{a}$

$$= \frac{e^{ax}\sin(bx)}{a} - \frac{b}{a}\left(+ \frac{e^{ax}\cos(bx)}{a}\cancel{\sin(bx)} - \int \frac{-b}{a} e^{ax}\sin b\right.$$

$$= \frac{e^{ax}\sin(bx)}{a} \cancel{+} - \frac{b}{a^2} e^{ax}\cos(bx)\,\cancel{\sin(bx)} - \frac{b^2}{a^2}\int e^{ax}\sin(bx)\,dx$$

$$\int e^{ax}\sin(bx)\,dx + \frac{b^2}{a^2}\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}\sin(bx)}{a} \cancel{+} - \frac{b}{a^2} e^{ax}\cos(bx)$$

$$\frac{a^2+b^2}{a^2}\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2}\left(a\sin(bx) + b\cos(bx)\right)$$

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}\left(a\sin(bx) + b\cos(bx)\right)}{a^2+b^2}$$