PREGUNTA 1

	PTJE
a	3
b	3,5
c	3,5

(a) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen}(a+x)-\operatorname{sen}(a-x)}{x} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{\cancel{\operatorname{sen}a\cos x} + \operatorname{sen}x\cos a - (\cancel{\operatorname{sen}a\cos x} - \operatorname{sen}x\cos a)}{x}$ (1P)

$\lim\limits_{x\to 0} 2\cos a \cdot \dfrac{\operatorname{sen}x}{x}$ (1P) $= 2\cos a \lim\limits_{x\to 0} \dfrac{\operatorname{sen}x}{x}^{1}$

$\therefore \lim\limits_{x\to 0} \dfrac{\operatorname{sen}(a+x)-\operatorname{sen}(a-x)}{x} = 2\cos a$ (1P)

(b) $\lim\limits_{x\to 0} \dfrac{\operatorname{Tg}x - \operatorname{sen}x}{x^3} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x}{\cos x} - \operatorname{sen}x}{x^3}$ (0,5P) $= \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x - \operatorname{sen}x\cos x}{\cos x}}{x^3} = \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x(1-\cos x)}{\cos x}}{x^3}$

$\lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x(1-\cos x)}{\cos x}\cdot\frac{1+\cos x}{1+\cos x}}{x^3}$ (0,5P) $= \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x\cdot(1-\cos^2 x)}{\cos x(1+\cos x)}}{x^3} = \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}x\cdot\operatorname{sen}^2 x}{\cos x(1+\cos x)}}{x^3}$ (0,5P)

$\lim\limits_{x\to 0} \dfrac{\operatorname{sen}x}{x}^{1} \cdot \dfrac{\operatorname{sen}x}{x}^{1} \cdot \dfrac{\operatorname{sen}x}{x}^{1} \cdot \dfrac{1}{\cos x\cdot(1+\cos x)}$ (1P) $= \dfrac{1}{1(1+1)} = \dfrac{1}{2}$

$\therefore \lim\limits_{x\to 0} \dfrac{\operatorname{Tg}x - \operatorname{sen}x}{x^3} = \dfrac{1}{2}$ (1P)

(c) $\lim\limits_{x\to \frac{\pi}{3}} \dfrac{2\cos^2 x - 5\cos x + 2}{2\cos^2 x + 3\cos x - 2} = \dfrac{2\overset{0,25}{\cos^2\frac{\pi}{3}} - 5\overset{0,5}{\cos\frac{\pi}{3}} + 2}{2\underset{0,25}{\cos^2\frac{\pi}{3}} - 3\underset{0,5}{\cos\frac{\pi}{3}} - 2}$ (1,5P) $= \dfrac{0}{-3} = 0$ (2P)

$\therefore \lim\limits_{x\to \frac{\pi}{3}} \dfrac{2\cos^2 x - 5\cos x + 2}{2\cos^2 x + 3\cos x - 2} = 0$

PREGUNTA 2

a	5P
B	5P

a) $\lim\limits_{x\to 0} \dfrac{1-\cos(ax)}{x^2} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{1-\cos ax}{x^2} \cdot \dfrac{1+\cos ax}{1+\cos ax}$ (1P) $= \lim\limits_{x\to 0} \dfrac{1-\cos^2 ax}{x^2(1+\cos ax)} = \lim\limits_{x\to 0} \dfrac{\sin^2 ax}{x^2(1+\cos ax)} \cdot \dfrac{a^2}{a^2}$ (1P)

$\lim\limits_{x\to 0} \dfrac{\sin \cancel{ax}}{\cancel{ax}}^{1} \cdot \dfrac{\sin \cancel{ax}}{\cancel{ax}}^{1} \cdot \dfrac{a^2}{1+\cos ax}$ (1P) $= \dfrac{a^2}{1+\cos(a\cdot 0)} = \dfrac{a^2}{2}$ (1P)

b) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin 2x} - \sqrt{1-\sin 3x}}{x} = \dfrac{0}{0}$

$\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin 2x} - \sqrt{1-\sin 3x}}{x} \cdot \dfrac{\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}}{\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}}$ (1P)

$\lim\limits_{x\to 0} \dfrac{1+\sin 2x - (1-\sin 3x)}{x\left(\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}\right)} = \lim\limits_{x\to 0} \dfrac{\sin 2x + \sin 3x}{x\left(\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}\right)}$ (1P)

$\lim\limits_{x\to 0} \left(\dfrac{\sin 2x}{x} \cdot \dfrac{2}{2} + \dfrac{\sin 3x}{x} \cdot \dfrac{3}{3}\right) \cdot \dfrac{1}{\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}}$ (1P)

$\lim\limits_{x\to 0} \left(2\, \dfrac{\sin \cancel{2x}}{\cancel{2x}}^{1} + 3\cdot \dfrac{\sin \cancel{3x}}{\cancel{3x}}^{1}\right) \cdot \lim\limits_{x\to 0} \dfrac{1}{\sqrt{1+\sin 2x} + \sqrt{1-\sin 3x}}$ (1P)

$5 \cdot \dfrac{1}{2} = \dfrac{5}{2}$

$\therefore \lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin 2x} - \sqrt{1-\sin 3x}}{x} = \dfrac{5}{2}$ (1P)

PREGUNTA 3 :

a	3P
b	2P
c	5P.

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$

$\lim_{n\to 0} (1+n)^{\frac{1}{n}} = e$

(a) $\lim_{n\to\infty} \left(\frac{n+2}{n-3}\right)^{\frac{2n-1}{5}}$

$\lim_{n\to\infty} \left(\frac{n-3+3+2}{n-3}\right)^{\frac{2n-1}{5}} = \lim_{n\to\infty} \left(\cancel{\frac{n-3}{n-3}}^{1} + \frac{5}{n-3}\right)^{\frac{2n-1}{5}} =$

$\lim_{n\to\infty} \left(1+\frac{1}{\frac{n-3}{5}}\right)^{\frac{2n-1}{5}}$ (1P) $= \lim_{n\to\infty} \left(\cancel{\left(1+\frac{1}{\frac{n-3}{5}}\right)^{\frac{n-3}{5}}}\right)^{\frac{2n-1}{5} : \frac{n-3}{5}}$ (1P)

$e^{\lim_{n\to\infty} \frac{2n-1}{n-3} : \frac{n}{n}} = e^{\lim_{n\to\infty} \frac{2-\cancel{\frac{1}{n}}^{0}}{1-\cancel{\frac{3}{n}}^{0}}} = e^{2}$ (1P) $\to \therefore \lim_{n\to\infty} \left(\frac{n+2}{n-3}\right)^{\frac{2n-1}{5}} = e^{2}$

(b) $\lim_{x\to 0} (1+2x)^{\frac{3}{7x}} = \lim_{x\to 0} \left(\underbrace{\cancel{(1+2x)^{\frac{1}{2x}}}}_{e}\right)^{2\cancel{x}\cdot\frac{3}{7\cancel{x}}}$ (1P) $= e^{\frac{6}{7}}$ (1P) $\to \therefore \lim_{x\to 0} (1+2x)^{\frac{3}{7x}} = e^{\frac{6}{7}}$

(c) $\lim_{x\to -1} \left(\frac{2x+5}{x+4}\right)^{\frac{2}{x+1}} =$ Cambio de variable

$u = x+1 \to x = u-1$ (1P)

$u = -1+1 \to u \to 0.$

$\lim_{u\to 0} \left(\frac{2(u-1)+5}{u-1+4}\right)^{\frac{2}{u}}$ (1P) $= \lim_{u\to 0} \left(\frac{2u+3}{u+3}\right)^{\frac{2}{u}} = \lim_{u\to 0} \left(\frac{u+3+u}{u+3}\right)^{\frac{2}{u}}$

$\lim_{u\to 0} \left(\cancel{\frac{u+3}{u+3}}^{1} + \frac{u}{u+3}\right)^{\frac{2}{u}\cdot\frac{1}{\frac{u}{u+3}}\cdot\frac{u}{u+3}}$ (1P) $= \lim_{u\to 0} \left(\underbrace{\cancel{\left(1+\frac{u}{u+3}\right)^{\frac{1}{\frac{u}{u+3}}}}}_{e}\right)^{\frac{2}{\cancel{u}}\cdot\frac{\cancel{u}}{u+3}}$ (1P)

$e^{\lim_{u\to 0} \frac{2}{u+3}} = e^{\frac{2}{3}}$ (1P)

$\therefore \lim_{x\to -1} \left(\frac{2x+5}{x+4}\right)^{\frac{2}{x+1}} = e^{\frac{2}{3}}$

PREGUNTA 4 :

a	3 P
b	3 P
C	4 P

ⓐ $\lim_{x \to a} f(x)$ $\quad b - [x - a] \leq f(x) \leq b + [x - a]$

Por el método del ENCAJE

$$g x \leq f(x) \leq I(x)$$

$$\lim_{x \to c} g(x) = \lim_{x \to c} I(x) = \lim_{x \to c} f(x)$$

$$b - [x - a] \leq f(x) \leq b + [x - a]$$

$\lim_{x \to a} [b - [x - a]]$ $\qquad \lim_{x \to a} b + [x - a]$

$\lim_{x \to a} b - [x - a] = b - [a - a]$ $\qquad \lim_{x \to a} b + [x - a] = b + [a - a]$

$\lim_{x \to a} b - [x - a] = b$ (1P) $\qquad \lim_{x \to a} b + [x - a] = b$ (1P)

$$\therefore \lim_{x \to a} b - [x - a] = \lim_{x \to a} b + [x - a] = b$$

$\lim_{x \to a} f(x) = b$ (1P)

ⓑ $\lim_{x \to \infty} \frac{3x^4 - 2x + 1}{3x^2 + 6x - 2} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{3x^4 - 2x + 1}{3x^2 + 6x - 2} : \frac{x^4}{x^4}$ (1P) $= \lim_{x \to \infty} \frac{3 - \cancel{\frac{2}{x^3}}^{0} + \cancel{\frac{1}{x^4}}^{0}}{\cancel{\frac{3}{x^2}}^{0} + \cancel{\frac{6}{x^3}}^{0} - \cancel{\frac{2}{x^4}}^{0}} = \frac{3}{0}$ (1P) $= \nexists \lim_{x \to \infty} f(x)$ (1P)

ⓒ $\lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}$ (1P) $= \lim_{h \to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h}$ (0,5P) $= \lim_{h \to 0} \ln\left(\frac{x+h}{x}\right)^{\frac{1}{h}}$ (0,5P)

$\ln \lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{\frac{1}{h} : \frac{x}{x}} = \ln \underbrace{\lim_{h \to 0} \left(1 + \frac{h}{x}\right)^{\frac{1}{\frac{h}{x}}}}_{e} \cdot \frac{1}{x}$ (1P) $= \ln e^{\frac{1}{x}}$ (0,5P) $= \frac{1}{x} \cancel{\ln e} = \frac{1}{x}$

$$\therefore \lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h} = \frac{1}{x} \text{ (0,5P)}$$

SÓLO SE REVISAN 2 PREGUNTAS.
10 P.T S.

a	5P
b	5P
c	5P

PREGUNTA 5

(a) $\lim_{x\to 3} \frac{x^3 - 3x^2 + 2x - 6}{x-3} = \frac{0}{0}$

$\lim_{x\to 3} \frac{x^2(x-3) + 2(x-3)}{x-3}$ (2P) $= \lim_{x\to 3} \frac{\cancel{(x-3)}(x^2+2)}{\cancel{(x-3)}} = 11$ (2P)

$\therefore \lim_{x\to 3} \frac{x^3 - 3x^2 + 2x - 6}{x-3} = 11$ (1P)

(b) $\lim_{x\to 2} \frac{x^2+3x-10}{3x^2-5x-2} = \frac{0}{0}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+5)}{3\cancel{(x-2)}(x+\frac{1}{3})}$ (3P.)

$3x^2 - 5x - 2$
$3(x^2 - \frac{5}{3}x - \frac{2}{3})$
$3((x-2)(x+\frac{1}{3}))$

$\lim_{x\to 2} \frac{(x+5)}{3(x+\frac{1}{3})} = \frac{7}{\cancel{3}\cdot\frac{7}{\cancel{3}}} = 1$

$\therefore \lim_{x\to 2} \frac{x^2+3x-10}{3x^2-5x-2} = 1$ (2P)

(c) $\lim_{x\to 2} \frac{1}{x^2-2x} - \frac{x}{x-2} =$

$\lim_{x\to 2} \frac{1}{x(x-2)} - \frac{x}{(x-2)} = \lim_{x\to 2} \frac{1-x^2}{x(x-2)}$ (2P.) $= \lim_{x\to 2} \frac{(1-x)(1+x)}{x(x-2)} = \frac{-3}{0}$ (1P)

$\therefore \lim_{x\to 2} \frac{1}{x^2-2x} - \frac{x}{x-2} = \nexists \lim_{x\to 2} f(x)$ (2P)

PREGUNTA 6

a	5P
b	5P.

(a) $\lim_{x\to 1} \frac{\sqrt[3]{x}-x}{\sqrt{x}-x} = \frac{0}{0}$

$\lim_{x\to 1} \frac{\sqrt[3]{x}-x}{\sqrt{x}-x} \cdot \frac{(\sqrt{x}+x)\cdot(\sqrt[3]{x}^2+\sqrt[3]{x}\cdot x+x^2)}{(\sqrt{x}+x)(\sqrt[3]{x}^2+\sqrt[3]{x}\cdot x+x^2)}$ (1P) $= \lim_{x\to 1} \frac{(x-x^3)\cdot(\sqrt{x}+x)}{(x-x^2)(\sqrt[3]{x}^2+\sqrt[3]{x}\,x+x^2)}$ (1P)

$\lim_{x\to 1} \frac{\cancel{x}(\cancel{1-x})(1+x)(\sqrt{x}+x)}{\cancel{x}(\cancel{1-x})(\sqrt[3]{x^2}+\sqrt[3]{x}\,x+x^2)}$ (2P) $= \frac{2\cdot 2}{1+1+1} = \frac{4}{3}$

$\therefore \lim_{x\to 1} \frac{\sqrt[3]{x}-x}{\sqrt{x}-x} = \frac{4}{3}$ (1P)

(b) $\lim_{x\to 0} \frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} = \frac{0}{0}$

$\lim_{x\to 0} \frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} \cdot \frac{(\sqrt{x^2+a^2}+a)\cdot(\sqrt{x^2+b^2}+b)}{(\sqrt{x^2+a^2}+a)(\sqrt{x^2+b^2}+b)}$ (2P)

$\lim_{x\to 0} \frac{(x^2+\cancel{a^2}-\cancel{a^2})(\sqrt{x^2+b^2}+b)}{(x^2+\cancel{b^2}-\cancel{b^2})(\sqrt{x^2+a^2}+a)} = \lim_{x\to 0} \frac{\cancel{x^2}(\sqrt{x^2+b^2}+b)}{\cancel{x^2}(\sqrt{x^2+a^2}+a)}$ (1P) $= \frac{\cancel{2}b}{\cancel{2}a} = \frac{b}{a}$

$\therefore \lim_{x\to 0} \frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b} = \frac{b}{a}$ (2P)