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Guía N°1: Aplicación de los limites fundamentales

Calcule los siguientes límites:

1) $\lim_{n\to\infty} \frac{2n^3+3n+1}{3n^3+5n^2+n}$ R: 2/3

2) $\lim_{n\to\infty} \frac{n}{\sqrt[3]{n^3+10}}$ R: 1

3) $\lim_{n\to\infty} \sqrt{n^2+1} - n$ R: 0

4) $\lim_{a\to\infty} \frac{a}{3^a}$ R: 0

5) $\lim_{b\to\infty} \frac{b^2+1}{b^3-2}$ R: 0

6) $\lim_{n\to\infty} \frac{(-4)^n+5^n}{(-4)^{n+1}+2*5^n}$ R: $\frac{1}{2}$

7) $\lim_{n\to\infty} \sqrt{n} - \sqrt{n+1}$ R: 0

8) $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$ R: $\frac{1}{e}$

Resolución Guía 1.

(1) $$\lim_{n\to\infty} \frac{2n^3+3n+1}{3n^3+5n^2+n}$$

$$= \lim_{n\to\infty} \left[\frac{2n^3+3n+1}{3n^3+5n^2+n} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}\right]$$

$$= \lim_{n\to\infty} \frac{2+\cancel{\frac{3}{n^2}}^{0}+\cancel{\frac{1}{n^3}}^{0}}{3+\cancel{\frac{5}{n}}^{0}+\cancel{\frac{1}{n^2}}^{0}}$$

$$= 2/3$$

(2) $$\lim_{n\to\infty} \frac{n}{\sqrt[3]{n^3+10}}$$

$$= \lim_{n\to\infty} \left[\frac{n}{\sqrt[3]{n^3+10}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right]$$

$$= \lim_{n\to\infty} \frac{\cancel{\frac{n}{n}}^{1}}{\sqrt[3]{\cancel{\frac{n^3}{n^3}}^{1}+\cancel{\frac{10}{n^3}}^{0}}}$$

$$= 1$$

(3) $$\lim_{n\to\infty} \sqrt{n^2+1}-n$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2+1}-n \cdot \frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}\right]$$

$$= \lim_{n\to\infty} \frac{\left(\sqrt{n^2+1}\right)^2-(n)^2}{\sqrt{n^2+1}+n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}+1-\cancel{n^2}}{\sqrt{n^2+1}+n}$$

$$= \lim_{n\to\infty} \frac{1}{\cancel{\sqrt{n^2+1}+n}^{\infty}}$$

$$= 0$$

(4) $$\lim_{a \to \infty} \frac{a}{3^a}$$

pensando en mi mente

$a=1 \quad \frac{1}{3} = 0{,}3\overline{3}$

$a=2 \quad \frac{2}{9} = 0{,}2\overline{2}$

$a=3 \quad \frac{3}{27} = 0{,}1\overline{1}$

$\vdots$

$a=10 \quad \frac{10}{3^{10}} = 1{,}69 \cdot 10^{-4}$

$\therefore$ tiende a $\underline{0}$

$$= \lim_{a \to \infty} \frac{a}{3^a}$$

$$= 0$$

(5) $$\lim_{b \to \infty} \frac{b^2+1}{b^3-2}$$

$$= \lim_{b \to \infty} \left[\frac{b^2+1}{b^3-2} \cdot \frac{\frac{1}{b^3}}{\frac{1}{b^3}} \right]$$

$$= \lim_{b \to \infty} \frac{\cancel{\frac{1}{b}}^{0} + \cancel{\frac{1}{b^3}}^{0}}{1 - \cancel{\frac{2}{b^3}}^{0}}$$

$$= 0$$

(6) $$\lim_{n \to \infty} \frac{(-4)^n + 5^n}{(-4)^{n+1} + 2 \cdot 5^n}$$

$$= \lim_{n \to \infty} \left[\frac{(-4)^n + 5^n}{(-4)^n (-4) + 2 \cdot 5^n} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} \right]$$

$$= \lim_{n \to \infty} \frac{\frac{(-4)^n}{5^n} + 1}{\frac{(-4)^n(-4)}{5^n} + 2}$$

$$= \lim_{n \to \infty} \frac{\cancel{\left(\frac{-4}{5}\right)^n}^{0} + 1}{\cancel{\left(\frac{-4}{5}\right)^n \cdot 4}^{0} + 2}$$

$$= 1/2$$

(7) $\lim\limits_{n \to \infty} \sqrt{n} - \sqrt{n+1}$

$$= \lim_{n \to \infty} \left[\sqrt{n} - \sqrt{n+1} \cdot \frac{\sqrt{n} + \sqrt{n+1}}{\sqrt{n} + \sqrt{n+1}} \right]$$

$$= \lim_{n \to \infty} \frac{n - (n+1)}{\sqrt{n} + \sqrt{n+1}}$$

$$= \lim_{n \to \infty} \frac{-1}{\cancel{\sqrt{n} + \sqrt{n+1}}} \; 0$$

$$= 0$$

(8) $\lim\limits_{n \to \infty} \left(\frac{n}{n+1} \right)^{n}$

$$= \lim_{n \to \infty} \left[\left(\frac{n}{n+1} \right)^{-1} \right]^{\frac{n}{-1}}$$

$$= \lim_{n \to \infty} \left[\left(\frac{n+1}{n} \right)^{n} \right]^{\frac{1}{-1}}$$

$$= e^{-1} = 1/e$$